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University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE AND SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE IN INFORMATION TECHNOLOGY

SMA 2101: CALCULUS I

DATE: AUGUST 2013 **TIME: 2HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) Evaluate the following limits

(i) $$\lim_{x-\infty} \frac{3x-2}{\sqrt{2x^2+1}}$$ (4 Marks)

(ii) $$\lim_{x\to 2} \left\{ \frac{\sqrt{x^2+5}-3}{x^2-2x} \right\}$$ (4 Marks)

(b) Find the relationship between a and b so that the function f defined by

$f(x) \begin{cases} ax+1, & if \quad x \le 3 \\ bx+3, & if \quad x > 3 \end{cases}$ is continuous at $x = 3$ (4 Marks)

(c) Use the definition of derivative to obtain the derivative of the function

$f(t) = \frac{2}{t}$ (3 Marks)

(d) Find the derivatives of the following functions

(i) $z = \left(\frac{2t+5}{t^2+1}\right)^4$ (3 Marks)

(ii) $y = (x^2+1)^{\sin x}$ (3 Marks)

(iii) $y = x \sin^{-1}(x^2)$ (3 Marks)

(e) Use derivatives to estimate the value of $\sqrt[3]{65}$ (3 Marks)

(f) A spherical balloon is blown up so that its volume increases at a constant rate of $2cm^3$ per second. Find the rate of increase of its radius when its volume is $50cm^3$.
(3 Marks)

QUESTION TWO – (20 MARKS)

(a) Find $\frac{dy}{dx}$ for $y = \tan^{-1}\sqrt{x+1}$ (3 Marks)

(b) A closed cylindrical metal tin is to have a capacity of $250\,\pi\, ml$. If the area of the metal used is to be a minimum, what should the radius of the tin be. (4 Marks)

(c) Given that $x = \theta - \sin\theta,\ y = 1 - \cos\theta$, find $\frac{d^2y}{dx^2}$ (4 Marks)

(d) Find the equation of the tangent to the curve $y^3 - xy^2 + \cos xy = 2$ at the point $(0,1)$.
(4 Marks)

(e) From first principle find $\frac{dy}{dx}$ given $y = \frac{3-4x}{1-2x}$ (5 Marks)

QUESTION THREE – (20 MARKS)

(a) Find $\frac{dy}{dx}$ if $y = \ln\left\{\sqrt{\frac{x-1}{x^2}}\right\}, x > 1$ (3 Marks)

(b) Sketch the curve of the function $y = 5x^4 - x^5$ (6 Marks)

(c) Find all points of discontinuity of f, where f is defined by: (7 Marks)

$$f(x) = \begin{cases} 2x+3, & if \quad x \le 2 \\ 2x-3, & if \quad x > 2 \end{cases}$$

(d) Find $\frac{dy}{dx}$ at $(1,1)$ given $x^2 \ln(xy^2) = 10(x - y^3)$. (4 Marks)

QUESTION FOUR – (20 MARKS)

(a) If $y = \frac{\cos x}{x}$, prove that $\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} + y = 0$ (5 Marks)

(b) Show that function $f(x) = |x|$ is differentiable everywhere except at $x = 0$. Illustrate the conclusion of derivative . (5 Marks)

(c) Find the derivative of $f(x) = \sec x$ using the definition of derivative. (4 Marks)

(d) A closed rectangular container has a square base and is required to have a volume of $64cm^3$. If the container is made of thin metal, find the dimensions which will minimize the surface area. (6 Marks)