CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRSTY YEAR EXAMINATIONS FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

COMP 102: DISCRETE MATHEMATICS

STREAMS: BSC (COMP SCIENCE) Y1S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 21/4/2015 **8.30 A.M – 10.30 A.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** questions.
- Diagrams should be used whenever they are relevant to support an answer.
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely
- Electronic, non-programmable calculators may be used

SECTION A: COMPULSORY

QUESTION 1[30 MARKS]

a) What is the definition of a *proposition*?, give an example [4 marks]

b) Which of the following sentences are propositions? Are they true or false?

(i) $x+y=y+x$. [2 marks]

(ii) $x^2+3=7$ [2 marks]

(iii) Solve these problems.

(iv) For every positive real number x, $x+\frac{1}{x}\geq 2$ [2 marks]

(v) Every prime number is odd. [2 marks]

c) Write the *negation* of each of the following propositions without using any form of the word "not". For example, the negation of "She gets up before noon," is "She gets up at noon or later."

(i) Today is Monday. [2 marks]

(ii) $3 + 4 = 10$ [2 marks]

(iii) The weather in Chuka is cold and dry. [2 marks]

d) Use a *truth table* to determine whether the followings are a tautology, a contradiction, or a contingency.

(i) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ [2 marks]

(ii) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow \neg r$ [2 marks]

e) Show that the following pairs of propositions are *logically equivalent* **without** using a truth table.

(i) $\neg(p \wedge (\neg p \vee q))$ and $\neg p \vee \neg q$ [4 marks]

(ii) $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $p \leftrightarrow q$ [4 marks]

SECTION B: ANSWER ONLY TWO QUESTIONS FROM THIS SECTION

Question 2 [20 marks]

How many students are enrolled in a course either in calculus, discrete mathematics, data structures, or programming languages at a school if there are 507, 292, 312, and 344 students in these courses, respectively; 14 in both calculus and data structures; 213 in both calculus and programming languages; 211 in both discrete mathematics and data structures; 43 in both discrete mathematics and programming languages; and no student may take calculus and discrete mathematics, or data structures and programming languages concurrently? Show your working clearly. [20 marks]

Question 3 [20 marks]

The symmetric difference of sets A and B, written $A \oplus B$, is defined as follows:

$A \oplus B = \{x \in U : (x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)\}$

(U is the universal set.) Prove that $A \oplus B \subseteq (A - B) \cup (B - A)$.

Question 4[20 marks]

Let p, q, and r be the propositions

p: You get an A on the final exam

q: You do every set of homework of this class

r: You get an A in this class

f) Write the following propositions using p, q, and r and logical connectives [10 marks]

(i) To get an A in this class, it is necessary for you to get an A on the final.

(ii) Getting an A on the final and doing every set of homework of this class is sufficient for getting an A in this class.

(iii) You get an A on the final, but you don't do every set of homework of this class; nevertheless, you get an A in this class.

g) Translate the following propositions into English sentences. [10 marks]

(i) $r \leftrightarrow p \vee q$

(ii) $\neg(p \wedge q) \rightarrow \neg r$

(iii) $\neg((p \wedge q) \rightarrow r)$

Question 5 [20 marks]

h) **Island of Questioners:** On the Island of Questioners, inhabitants never make statements. They only ask questions answerable by **Yes** or **No**. Each inhabitant is one of two types, **A** and **B**. Those of type A ask only questions whose correct answer is Yes. Those of type B ask only questions whose correct answer is No. For example, an inhabitant of type A could ask, "Does two plus two equal four?" But he could not ask whether two plus two equals five. An inhabitant of type B could not ask whether two plus two equals four, but he could ask whether two plus two equals five.

Muchiri and Chebet are two inhabitants on the island. One day Muchiri asks someone, "are Chebet and I both of type B?" What type is Chebet? Show your reasoning. **[10mks]**

i) **Knight or Knave:** Joan is either a knight or a knave (not both). Knights always tell the truth, and only the truth; Knaves always tell lies, and only lies. Someone asks Joan, "Are you a knight?" She replies, "If I am a knight then I will eat my hat."

Will Joan eat her hat? Why or why not? Show your working **[10mks]**
