KISII UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF THE DEGREE OF BACHELOR OF EDUCATION [SCIENCE]/ACTUARIAL SCIENCE/ECONOMICS AND STATISTICS/ANALYTICAL CHEMISTRY/APPLIED STATISTICS AND COMPUTER SCIENCE
SECOND SEMESTER 2017/2018
[MAY – AUGUST, 2017]

MATH 241: PROBABILITY AND STATISTICS II

STREAM: Y2S2 **TIME: 2 HOURS**

DAY: THURSDAY, 9:00-11:00 AM **DATE: 14/09/2017**

INSTRUCTIONS

1. ***Do not write anything on this question Paper.***
2. ***Answer question ONE and any other TWO questions.***
3. ***Show all your workings and steps.***

QUESTION ONE [COMPULORY (30marks)]

(a) Determine the χ^2 percentile that is required to construct each of the following confidence intervals.

(i) Confidence level =95%, degreees of freedom=24, one-sided (upper)

(ii) Confidence level =99%, degreees of freedom=9, one-sided (lower)

(iii) Confidence level =90%, degreees of freedom=19, two-sided

(6marks)

(b) (i) Let x be a continuous random variable with a p.d.f given by:

$$f(x)=\begin{cases}\frac{x}{18}, & 0\le x\le 6\\ 0, & otherwise\end{cases}$$

Find the p.d.f of the random variable y if $Y=2X+10$ **(3marks)**

(ii) Let X have a geometric probability distribution given by

$$f(x)=\begin{cases}p(1-p)^{x-1}, & x=1,2,\ldots\\ 0, & otherwise\end{cases}$$

Find the probability density function of $Y=X+2$ **(3marks)**

(c) (i) State and prove Bonferroni's inequality. **(3marks)**

(ii) Suppose that we have ten events A_i (where $i=1,2,3,\ldots,10$) with $P(A_i)=0.99$. Estimate the joint probability $P(A_1\cap A_2\ldots A_{10})$. **(3marks)**

(d) The joint density function of two random variables X and Y given by:

$$f(x,y)=\begin{cases}\frac{xy}{4}, & 0\le x\le 2, 0\le y\le 2\\ 0, & elsewhere\end{cases}$$

Find the Joint density function of U and V if $U=X+2Y$

(6marks)

(e)) Let the p.d.f of the random variable X be defined as:

$$f(x)=\begin{cases}p^x(1-p)^{1-x}, & x=0,1\\ 0, & elsewhere.\end{cases}$$

Find the characteristic function of a random variable $Y=X_1+X_2+\ldots+X_n$ if the X_i's are independent. **(6marks)**

QUESTION TWO (20marks)

Let X_1 and X_2 be two stochastically independent random variables that have Gamma distributions and joint p.d.f given by

$$f(x_1,x_2)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}x_1^{\alpha-1}x_2^{\beta-1}e^{-x_1-x_2}, & x_1>0;x_2>0\\ 0, & otherwise\end{cases}$$

. Where $\alpha>0$, $\beta>0$. Let

$Y_1=X_1+X_2$ and $Y_2=\dfrac{X_1}{X_1+X_2}$. Show that Y_1 and Y_2 are stochastically independent. Hence determine the marginal probability functions of Y_1 and Y_2

(20marks)

QUESTION THREE (20marks)

(a) Let X_1 and X_2 have the joint probability distribution defined as:

$$f(x_1,x_2)=\begin{cases}\dfrac{x_1x_2}{36}, & x_1=1,2,3;x_2=1,2,3\\ 0, & otherwise\end{cases}$$

. Find the marginal probability density function of Y_1 if $Y_1=X_1X_2$ and $Y_2=X_2$.

(10marks)

(b) If $f(x_1,x_2)=\begin{cases}\begin{pmatrix}2\\x_1+x_2\end{pmatrix}\left(\dfrac{2}{3}\right)^{x_1+x_2}\left(\dfrac{1}{3}\right)^{2-x_1-x_2}, & x_1=0,1;x_2=0,1\\ 0, & otherwise\end{cases}$ is the joint density of X_1 and X_2, find the joint probability density function of $Y_1=X_1-X_2$ and $Y_2=X_1+X_2$. Hence determine the respective marginal probability density functions of Y_1 and Y_2.

(10marks)

QUESTION FOUR (20 marks)

(a) Show that the characteristic function of a random variable X having a Normal distribution with parameters μ, σ^2 is given by

$\phi(t) = e^{it\mu - \frac{1}{2}t^2\sigma^2}$ **(12marks)**

(b) Let $\bar{X}$ and S^2 be the mean and the variance of a random sample of size 25 from a distribution which is $N(3,100)$. Evaluate

$P(0 < \bar{X} < 6, 55.2 < S^2 < 145.6)$.

(8marks)

QUESTION FIVE (20 marks)

(a) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5 < Y_6$ denote the order statistics of a random sample of size 6 from a distribution having p.d.f $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$

Find:

(i) The p.d.f of the first order statistic.

(ii) The p.d.f of the sixth order statistic.

(iii) $P(0.5 < Y_6)$ **(12marks)**

(b) Let $Y_1 < Y_2 < Y_3$ be the order statistics of a random sample of size 3 from a distribution having p.d.f $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$, Find :

(i) The joint p.d.f of Y_1 and Y_3

(ii) The p.d.f of the range $Z_1 = Y_3 - Y_1$ **(8marks)**