## University Examinations 2010/2011

## SECOND YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF COMMERCE

## HBC 2122: OPERATIONS RESEARCH

INSTRUCTIONS: Answer question one and any other two questions

## QUESTION ONE - (30 MARKS)

(a) Define linear programming and state three areas where linear programming has been used successfully to solve practical problems in business.
(b) (i) Explain the concept of degeneracy in linear programming
(ii) Consider LPP

Maximize $z=3 x+9 y$
Subject to

$$
\begin{gathered}
x+4 y \leq 8 \\
x+2 y \leq 4 \\
x, y \geq 0
\end{gathered}
$$

Set up the starting simplex tableau and show that the starting basic solution is degenerate.
(6 Marks)
(c) (i) State the fundamental theorem of duality
(ii) Given the primal problem

$$
\begin{array}{ll}
\text { Maximize } & 6 x_{1}+5 x_{2}-3 x_{3}+4 x_{4} \text { subject to; } \\
& x_{2}+x_{3}+x_{4} \leq 8 \\
& x_{1}+x_{2}+2 x_{3}+3 x_{4} \leq 15 \\
& x_{1}+2 x_{2} \leq 10 \\
& 2 x_{1}+3 x_{2}+2 x_{3}-4 x_{4} \leq 24 \\
& -2 x_{1}-3 x_{2}-2 x_{3}+4 x_{4} \leq-24 \\
& x_{1}, x_{2}, x_{3}, x_{4} \geq 0
\end{array}
$$

Write down its dual problem
(6 Marks)
(d) Explain the following terms as used in network analysis.
(i) A path
(ii) A loop
(iii)Connected net work
(iv) A tree

## QUESTIN TWO - (20 MARKS)

(a) What is game theory
(b) Explain the following terms as used in game theory
(i) Pure strategy
(ii) Mixed strategy
(iii)Optimal strategy
(iv) Value of the game
(c) Describe five limitations of games in competition.

QUESTION THREE = (20 MARKS $)$
(a) (i) State the characteristics of a linear program in canonical form
(ii) Convert the linear program below to canonical form.

$$
\begin{array}{ll}
\text { Minimize } & z=3 x+4 y \\
\text { Subject to } & x+2 y \geq 12 \\
& 2 x-3 y \leq 18 \\
& x, y \geq 0
\end{array}
$$

(b) Machine time available on two machines A and B is to be allocated to production of some quantity of two products 1 and 2. The two machines A and B have 80hrs and 60hrs of time available on them respectively. the two products 1 and 2 require for their production different amounts of time on each of the machines as shown in the table below.

| Product | Time on machine A | Time on machine B |
| :--- | :--- | :--- |
| 1 | 2 hrs | 3 hrs |
| 2 | 4 hrs | 2 hrs |
|  | 80 hrs | 60 hrs |

Each unit of product 1 is sold at Ksh60 and each unit of product 2 is sold at Ksh. 50. Formulate a linear programming model that will maximize the production and apply graphic method to solve the problem.

## QUESTION FOUR - (20 MARKS)

(a) Convert the following linear programs to standard form.
(i) Minimize $z=3 x+4 y$

Subject to

$$
\begin{align*}
& 3 x+y \leq 34 \\
& -x+2 y \geq 12 \\
& x, y \geq 0 \tag{3Marks}
\end{align*}
$$

(ii) Maximize $\quad z=4 x_{1}+3 x_{2}$

Subject to $\quad x_{1}+x_{2} \leq 40$
$2 x_{1}+x_{2} \leq 60$
$x_{1}, x_{2} \geq 0$
(b) A firm manufactures two types of bearings, A and B each of which requires processing time on lathes, grinders and polishers. The machine times needed for each type of bearing are given in the table

| Bearing |  | Time required in (hours) |  |  |
| :--- | :--- | :--- | :--- | :---: |
| type | Lathe | Grinder | polisher |  |
|  | 2 | 8 | 5 |  |
| A | 5 | 5 | 2 |  |
| B |  |  |  |  |

The total machine time available is 250 hours on lathes, 310 hours on grinders and 160 hours on polishers. The net profit per bearing of type $A$ is $£ 9$ and of type $B$ is $£ 10$. Apply the simplex Algorithm to determine the number of each type to be produced to maximize profit and state the maximum profit.

## QUESTION FIVE - (20 MARKS)

A distribution system has the following constraints

| FACTORY | CAPACITY (UNITS) |
| :--- | :--- |
| A | 45 |
| B | 15 |
| C | 40 |
|  |  |
| WAREHOUSE | DEMAND (UNITS) |
| A | 25 |
| B | 55 |
| C | 20 |

The transportation costs per unit (in Ksh) allocated with each route are as follows:
TO

|  | A | B | C |
| :--- | :--- | :--- | :--- |
| A | 10 | 7 | 8 |
| B | 15 | 12 | 9 |
| C | 7 | 8 | 12 |

Find the minimum cost of transportation using:
(i) The north west corner method
(6 Marks)
(ii) The least cost method
(iii)The Vogel's approximation method
(7 Marks)
(7 Marks

