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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS**

**1st YEAR 2nd SEMESTER 2016/2017ACADEMIC YEAR**

**MAIN REGULAR**

**COURSE CODE: SMA 822**

**COURSE TITLE: BANACH ALGEBRA I**

**EXAM VENUE: LR 1 STREAM: (Msc. Pure Mathematics)**

DATE: 24/04/17 EXAM SESSION: 11.30 – 2.30PM

TIME: 3.00 HOURS

**Instructions:**

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION ONE [20 MARKS]**

1. Define multiplicative linear functional and show that . (7 marks)
2. Define a -algebra giving two examples and show that a linear functional *f* is bounded with  (9 marks)
3. Describe a Gelfand-Naimark-Segal transform and hence show that it is a contractive Banach algebraic homomorphism. (3 marks)

**QUESTION TWO [20 MARKS]**

1. Analytically describe: A normed\*-algebra; sub-algebra and Banach algebra. (6 marks)
2. Prove that the intersection of two Banachalgebras is Banach algebra. (10 marks)
3. State and prove the condition under which the set of all bounded valued on nonempty set *S* is a unital Normed \*- algebra. (4 marks)

**QUESTION THREE [20 MARKS]**

1. Differentiate between the left inverse and right inverse in Banach algebras. (6 marks)
2. Show that an inverse in a Banach algebra is unique. (7 marks)
3. Let bea Banach algebra and its unity. Prove that if and then there existsuch that. (10 marks)

**QUESTION FOUR [20 MARKS]**

1. Describe: Trivial ideal, modula ideal, Maximal ideal and prime ideal. (8 marks)
2. Let  be an algebra and . Show that is a modula ideal. (8 marks)
3. Describe the process of unitization of normed algebras. (8 marks)

**QUESTION THREE [20 MARKS]**

1. Prove that if $Z$ is the set off all intergers with counting measure then is a Banach algebra. (8 marks)
2. Prove that any nonempty open subset of irreducible Banach algebra is dense and irreducible. Moreover, prove that if *Y* is a subset of a Banach algebra *X*, which is irreduciblein its induced sub-algebra then the closure of *Y* is also irreducible. (12 marks)