MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

SAC 303: ACTUARIAL MATHEMATICS II

Date: 10th December, 2016 Time: 12.00 -3.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.

Question 1 (30 marks)

(a) Show that $\mu_x^{(\tau)} = \Sigma\mu_x^{(j)}$ (6 marks)

(b) For a double decrement $\mu_x^{(1)} = 0.2, \mu_x^{(2)} = 0.6$ Calculate ${}_2q_x^{(2)}$

(6 marks)

(c) Given for a double decrement table, that

$q_{40}{}'^{(1)} = 0.02\ and\ q_{40}{}'^{(2)} = 0.04\ , Calculate\ q_{40}{}^{(\tau)}$

(d) Show that $\int \mu_{x+s}^{(\tau)} ds = \infty$ (6 marks)

(e) If 100,00 is paid as benefit if the death is natural and 200,000 if the death is accidental andbb

(i) $\mu_x^{(\tau)}(t) = 0.001.\quad t \geq 0$

(ii) $\mu_x^{(1)}(t) = 0.0002.\quad t \geq 0$ where $\mu_x^{(1)}$ is the force of decrement due to the death by accidental means

(iii) $\delta = 0.06$

Calculate the single benefit premium for this insurance

(6 marks)

Question 2 (20 marks)

(a) For a trible-decrement table, you are given

(i) $\mu_x^{(1)}(t) = b\,,$

(ii) $\mu_x^{(2)}(t) = b\,,$

(iii) $\mu_x^{(3)}(t) = 2b$

You are given that the probability (x) will exit group within 3 years due to decrement 1 is 0.00884. Compute (i) b

(ii) E(T)

(10 M ark)

(b) For students entering a college, you are given the following from a multiple decrement model

(i) 1000 students enter the college at t = 0

(ii) Students leave the college for failure (1) or all other reasons (2)

(iii) $\mu_x^{(1)}(t) = \mu \qquad 0 \leq t \leq 4$

$\mu_x^{(2)}(t) = 0.04 \qquad 0 \leq t \leq 4$

(iv) 48 students are expected to leave the college during their first year due to all causes

Calculate the expected number of students who will leave because of failure during their fourth year

(10 marks)

Question 3 (20 marks)

(a) An insurance policy issued to (50) will pay 40,000 upon death if it is accidental and occurs within 25 years.

- An additional benefit of 10,000 will be paid regardless of the time or cause of death.
- The force of accidental death at all ages is 0.01
- The force of death for all other causes is 0.05 at all ages

Find the single premium for this policy.

(12 marks)

(b) Show that $p_x{}'^{(j)} \leq p_x{}^{(j)}$

(8 marks)

Question 4 (20 marks)

(a) An employer provides his employees aged 62 the following one-year term benefits, payable at the of the year of decrement.

- 1 if the decrement results from cause 1
- 2 if the decrement results from cause 2
- 6 if the decrement results from cause 3

If their associated single decrement tables, all three decrement follow de Moivres Law with ω = 65. You are given i = 10%. Find the APV at age 62 of benefits.

(10 marks)

(b) If the force of mortality μ_{x+t} changes to $\mu_{x+t} - c$ where c is a positive constant, find the value of the value of c for which the probability that (x) will die within ayear will be halved. Express the answer in terms of q_x

(10 marks)

Question 5 (20 marks)

(a) A multiple decrement model with 2 causes of decrement has force of decrement given by

$$\mu_{x+t}^{(1)} = \frac{1}{100-(x+t)}$$

And

$$\mu^{(2)} = \frac{2}{100-(x+t)} \qquad t < 100 - x$$

If x = 50, obtain expressions for

f(t,j), g(t), h(j) and h(j/t)

(10 marks)

(b) Consider a random survivorship group consisting of two subgroups (1) of survivors of 1600 births; (2) the survivors of 540 persons joining 10 years later at age 10. An excerpt from the appropriate mortality table for both subgroups follows:

x	l_x
0	40
10	39
70	26

If Y_1 and Y_2 are the numbers of survivors to age 70 out of subgroups (1) and (2) Respectively, estimate a number c such that $Pr(Y_1 + Y_2 > c) = 0.05$. Assume the Lives are independent.