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University Examinations 2011/2012

FIRST YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2201: LINEAR ALGEBRA 1

DATE: APRIL 2012 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Determine k so that the vectors $\underset{\sim}{u} = (2,3k,-4,1,5)$ and $\underset{\sim}{v} = (6,-1,3,7,2k)$ are orthogonal. (2 Marks)

b) Determine whether or not the vectors $\underset{\sim}{u} = (6,2,3,4)$, $\underset{\sim}{v} = (0,5,-3,1)$ and $\underset{\sim}{w} = (0,0,7,-2)$ are independent. (4 Marks)

c) Let T:$\mathbb{R}^3 \to \mathbb{R}^3$be the linear transformation defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$. Find a basis and the dimension of the image of T.(4 Marks)

d) Find the angle between the vector $\underset{\sim}{u} = -\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\underset{\sim}{v} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. (4 Marks)

e) Find the distance between the parallel planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$. (4 Marks)

f) Show that $w = \{(x,y,z)|x+y+z=0\}$ is a subspace of $\mathbb{R}^3$. (4 Marks)

g) Determine whether or not the transformation T:$\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (x-y, y-z, 2x)$ in linear. (4 Marks)

h) Show that T:$\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (x+z, x-z, y)$ is invertible, hence find T^{-1}. (4 Marks)

QUESTION TWO (20 MARKS)

a) Find the equation of the plane passing through the points P(-4,-1,-1), Q(-2,0,1) and R(-1,-2,-3). (6 Marks)

b) Show that $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,) = (x-y, 2x = 3y, 4x)$ is a linear mapping. (6 Marks)

c) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$, find a basis and the dimension of

   i. The image of T. (4 Marks)

   ii. The Kernel of T. (4 Marks)

QUESTION THREE (20 MARKS)

a) Let W be a subspace of $\mathbb{R}^5$ spanned by (1,-2,0,0,3), (2,-5,-3,-2,6) (0,5,15,10,0) and (2, 6,18,8,6). Find the dimension and a basis of W. (6 Marks)

b) show that the vectors u=(1,2,3), v=(0,1,2) and w=(0,0,1) span $\mathbb{R}^3$. (5 Marks)

c) Let u and w be the following subspace of $\mathbb{R}^4$.

$u = \{(a,b,c,d) | b - 2c + d = 0\}$
$w = \{(a,b,c,d) | a = d, b = 2c\}$

Find the dimensions and a basis of

   i. u (3 Marks)

   ii. w (3 Marks)

   iii. $u \cap w$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) Use the vectors $\underset{\sim}{a} = \boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$, $\underset{\sim}{b} = 2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\underset{\sim}{c} = 3\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}$ to prove that $a \times (b+c) = a \times b + a \times c$. (6 Marks)

b) Write the vector $\underset{\sim}{v} = (1,-2,5)$ as a linear combination of the vector $e_1 = (1,1,1), e_2 = (1,2,3)$ and $e_3 = (2,-1,1)$. (6 Marks)

c) Let V be the vector space of polynomial of degree ≤ 3 over $\mathcal{R}$. Determine whether or not the polynomials $u = t^3 - 3t^2 + 5t + 1$, $v = t^3 - t^2 + 8t + 2$ and $w = 2t^3 - 4t^2 + 9t + 5$ are linearly dependent. (6 Marks)

d) Determine whether or not $u = \{(x,y,z) | xy = 0\}$ is a subspace of $\mathbb{R}^3$. (2 Marks)