MERU UNIVERSITY COLLEGE OF SCIENCE & TECHNOLOGY

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University Examinations 2012/2013

FIRST YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTURIAL SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2201: LINEAR ALGEBRA I

DATE: AUGUST 2012 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Determine k so that the vectors $\underset{\sim}{u} = (2,3k,-4,1,5)$ and $\underset{\sim}{v} = (6,-1,3,7,2k)$ are orthogonal. (2 Marks)

b) Determine whether or not the vectors $\underset{\sim}{u} = (6,2,3,4)$, $\underset{\sim}{v} = (0,5,-3,1)$ and $\underset{\sim}{w} = (0,0,7,-2)$ are independent. (4 Marks)

c) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$. Find a basis and the dimension of the image of T. (4 Marks)

d) Let V be the vector space of polynomials of degree ≤ 3 over $\mathcal{R}$. Determine whether or not the polynomials $u = t^3-3t^2+5t+1$, $v = t^3-t^2+8t+2$ and $w = 2t^3-4t^2+9t+5$ are linearly dependent. (5 Marks)

e) Write the vector $\underset{\sim}{v} = (1,-2,5)$ as a linear combination of the vectors $e_1 = (1,1,1), e_2 = (1,2,3)$ and $e_3 = (2,-1,1)$. (5 Marks)

f) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x,y,z) = (x-3y-2z, y-4z, z)$, determine whether T is invertible and if so find the inverse of T. (5 Marks)

g) Show that $\{(1,1,1),(0,1,1),(0,1,-1)\}$ span $\mathbb{R}^3$. (5 Marks)

QUESTION TWO (20 MARKS)

a) Let W be a subspace of $\mathbb{R}^5$ spanned by (1,-2,0,0,3), (2,-5,-3,-2,6), (0,5,15,10,0) and (2, 6,18,8,6). Find the dimension and a basis of W. (4 Marks)

b) Determine whether or not the following are subspaces of $\mathbb{R}^3$

  i. $w = \{(x,y,z) | x+y+z = 0\}$ (3 Marks)

ii. $u = \{(x, y, z) | xy = 0\}$ (2 Marks)

c) Show that the vectors u=(1,2,3), v=(0,1,2) and w=(0,0,1) span $\mathbb{R}^3$. (5 Marks)

d) Let u and w be the following subspace of $\mathbb{R}^4$.

$u = \{(a, b, c, d) | b - 2c + d = 0\}$

$w = \{(a, b, c, d) | a = d, b = 2c\}$

Find the dimensions and a basis of

i. u (2 Marks)

ii. w (2 Marks)

iii. $u \cap w$ (2 Marks)

QUESTION THREE (20 MARKS)

a) Find the distance between the parallel planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$. (4 Marks)

b) Find the equation of the plane passing through point P(-4,-1,-1), Q(-2,0,1) and R(-1,-2,-3). (6 Marks)

c) Show that T:$\mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (x - y, 2x + 3y, 4x)$ is a linear mapping. (4 Marks)

d) Let T:$\mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear mapping defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find the basis and the dimension of the

i. Image of T. (3 Marks)

ii. Kernel of T. (3 Marks)

QUESTION FOUR (20 MARKS)

a) Prove that the diagonals of a rhombus are perpendicular. (4 Marks)

b) Find the equation of the plane passing through the points P (1,2,-1), Q(2,3,1) and R(3,-1,2). (5 Marks)

c) Find the distance between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$. (3 Marks)

d) Find the parametric equations for the line of intersection of the planes $3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$. (5 Marks)

e) Find the angle between the vectors $\underset{\sim}{u} = -\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\underset{\sim}{v} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. (3 Marks)