

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2101/STA 2104: CALCULUS 1/CALCULUS FOR STATISTICS

DATE: APRIL 2012

TIME: 2 HOURS

INSTRUCTIONS: Answer question one and any other two questions

- Mathematical tables(clean) may be used.
- Non-programmable calculators can be used.

QUESTION ONE (30 MARKS)

- a) Distinguish between definite and indefinite integrals as used in calculus. (2 Marks)
- b) Determine the domain and range of the function $f(x) = \frac{2}{\sqrt{3x-1}}$. (3 Marks)
- c) Find the possible values of k given that f(k) = 64 where $f(x) = x^2 + 10x + 25$. (3 Marks)
- d) Evaluate the limit $\lim_{x \to \infty} \left(\frac{x^2 + 2x 3}{4x^3 3x^2 + 5} \right)$

e) Verify if (or not) the function g(x) is continuous at the point x=3, given that $g(x) = \begin{cases} 2x - 4, x \ge 3\\ x + 5, x < 3 \end{cases}$ (4 Marks)

f) Use the first principles to differentiate the function $y = \sqrt{3 - 2x}$ with respect to x.

(3 Marks)

- g) Compute $\frac{dy}{dx}$ given that $5xy^3 + 3x^2y 2x = 7$. (5 Marks)
- h) Identify the stationary points for the function $f(x) = 2x^3 + x^2 4x$. (4 Marks)

QUESTION TWO (20 MARKS)

a) Find the inverse function and hence compute the value of $f^{-1}(2)$ given that $f(x) = \frac{2x-3}{3x+4}$. (5 Marks)

- b) Given that the functions f(x) and g(x) are defined in the set of real numbers and that $f(x) = x^3 2$, g(x) = 2x + 5. Compute;
 - i. $(g \circ f)(x)$ (3 Marks)
 - ii. (fg)(x) (3 Marks)
- iii. (f + g)(2) (2 Marks) c) Evaluate the limits

i.
$$\lim_{x \to 5} \frac{5 - x^2 + 4x}{x - 5}$$
 (3 Marks)
ii.
$$\lim_{x \to 4} \frac{2 - \sqrt{x}}{x^2 - 16}$$
 (4 Marks)

QUESTION THREE (20 MARKS)

- a) State the conditions necessary for a function f(x) to be continuous at a point x=a.
- b) Compute $\frac{dy}{dx}$ given i. $y = \frac{\cos 3x}{e^x}$ (4 Marks)

ii.
$$y = 5x^3 + 1n(4x - 6)$$
 (4 Marks)

c) Find the equations of the tangent line and the normal line to the curve $8x^3 - y - 32x + 38 = 0$ at the point where x=3. (9 Marks)

QUESTION FOUR (20 MARKS)

- a) Compute d²y/dx² for the curves given by the equations
 i. x = t² + √t, y = 3t tan t (5 Marks)
 ii. y = x√2x 3 (6 Marks)
 b) Differentiate y = sec⁻¹ x with respect to x. (6 Marks)
- c) Outline how the second derivative is used to obtain optimal values of a function y = f(x). (3 Marks)