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University Examinations 2013/2014

SECOND YEAR, SECOND SEMESTER EXAMINATIONS FOR DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE/ BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2202: ALGEBRAIC STRUCTURES

DATE: DECEMBER 2013 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) Consider the function of $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Determine

(i) $f^{-1}[\{x: 4 \leq x \leq 25\}]$ (2 Marks)

(ii) $f^{-1}[-9]$ (2 Marks)

(b) Let $R = \{(1,1), \ (2,3), \ (3,2)\}$ be the relation on $x = \{1, 2, 3\}$. Determine whether R is

(i) Reflexive (2 Marks)

(ii) Symmetric (2 Marks)

(iii)Transitive (2 Marks)

(c) Show that the set $\mathbb{Z}$ of all integers under the binary operation $*$ defined by $a * b = a + b + 3, \forall\, a, \ b \in \mathbb{Z}$ is an abelian group. (5 Marks)

(d) Consider the binary operation table below defined over a set $S = \{a, b, c, d\}$

$*$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d	d	c	c	d

Determine whether $*$ in the table satisfies

(i) Commutative law (3 Marks)

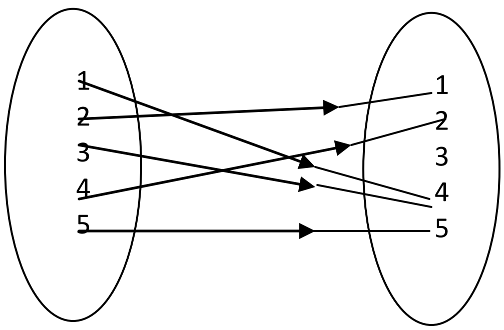
(ii) Associative law (3 Marks)

(e) Let f and g be two permutations defined on $S = \{a, b, c, d\}$ as, follows:

$$f = \begin{pmatrix} a & b & c & d \\ c & a & d & b \end{pmatrix} \text{ and } g = \begin{pmatrix} a & b & c & d \\ b & c & d & a \end{pmatrix}$$

Show that the composition of these two permutations is not commutative. (4 Marks)

(f) Let $A = \{1, 2, 3, 4, 5\}$ and $f: A \to A$ be defined by the diagram.



Find:

(i) $f[\{1, 3, 5\}]$ (1 Mark)

(ii) $f^{-1}[\{2, 3, 4\}]$ (2 Marks)

(iii) $f^{-1}[\{3,5\}]$ (2 Marks)

QUESTION TWO – (20 MARKS)

(a) Let $A = \{1, 2, 3, 4\}$, $B\{x, y, z, w\}$ and $C = \{5, 6, 7, 8\}$ and let
$u = \{(1, x),\ (1, y),\ (2, x),\ (3, w),\ (4, w)\}$ and $v = \{(y, 5),\ (y, 6),\ (z, 8),\ (w, 7)\}$
Determine VoU (7 Marks)

(b) Let R be the relation $<$ from $A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$.

(i) Write R as a set of ordered pairs. (2 Marks)

(ii) Plot R on a coordinate diagram of $A \times B$. (4 Marks)

(iii) Find the domain of R, range of R and R^{-1}. (3 Marks)

(iv) Find $R \circ R^{-1}$ (4 Marks)

QUESTION THREE – (20 MARKS)

(a) Prove that the set of non-zero integers modulo 6 under X_6 is not a group. (6 Marks)

(b) Show that the set G composed of $f_1, f_2, \ f_3, f_4$ of 4 transformations of the set of complex numbers in itself defined by $f_1(\mathbb{z}) = \mathbb{z}, \ f_2(\mathbb{z}) = -\mathbb{z}, \ f_2(\mathbb{z}) = \frac{1}{\mathbb{z}}, \ f_4(\mathbb{z}) = -\frac{1}{\mathbb{z}}, \forall\, \mathbb{z} \in c$ is an Abelian group with composite operation $i.e(G, \circ)$ where $G = \{f_1, f_2, f_3, f_4\}\ and\ f: \mathbb{c} \to \mathbb{c}$. (14 Marks)

QUESTION FOUR – (20 MARKS)

(a) Given that $\propto and\ \beta$ are two permutations of S_7, where

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix}\ and\ \ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 4 & 2 & 1 & 3 \end{pmatrix}$$

Find

(i) $(\propto \beta)^{-1}$ (3 Marks)

(ii) $\propto^{-1} \beta^{-1}$ (4 Marks)

(b) Find the group of symmetries of a regular hexagon. (6 Marks)

(c) Prove that $(H = \{0, 2, 4\}, +_6)$ is a subgroup of the group of integers modulo 6. (7 Marks)