MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

University Examinations 2012/2013

SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN
STATISTICS/BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER
SCIENCE AND BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SMA 2200/STA 2204: CALCULUS III/CALCULUS FOR STATISTICS

DATE: AUGUST 2013   **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Replace the Cartesian equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ by the equivalent polar equation.

(3 Marks)

b) Evaluate $\lim\limits_{x\to 0}(x^{\sin x})$ (3 Marks)

c) Given that $f(x) = x - x^3$. Find the extreme values of f on $[0, 1]$ and determine at which number in $[0,1]$ they occur. (4 Marks)

d) Evaluate the iterated integral

$$\int_{-2}^{3}\int_{x^2}^{x+6} xy\,dy\,dx$$
(4 Marks)

e) Find the Maclaurin's series generated by $f(x) = e^x$. (5 Marks)

f) Given that $F(x,y) = \sqrt{ln(4 - x^2 - y^2)}$. Find a function f of two variables and a function g of one variable such that $F = gof$. (3 Marks)

g) Given that $f(x,y) = \frac{x^3y - xy^3}{x^2+y^2}$, find $\frac{dy}{dx}$ and $\frac{df}{dy}$. (6 Marks)

h) State the mean value theorem. (2 Marks)

QUESTION TWO (20 MARKS)

a) Let $f(x,y) = 24xy - 6x^2y$. Find f_x and f_y and evaluate f_x and f_y at $(1,2)$.

(6 Marks)

b) Find the $\lim\limits_{x \to 0} \dfrac{e^x - x - 1}{x^2}$ (4 Marks)

c) Let $f(x) = \dfrac{1}{3}x^3 + 2x$. Find a number c in (0,3) such that $f'(c) = \dfrac{f(3 - f(0))}{3 - 0}$. (6 Marks)

d) Determine whether Rolle's theorem holds for $f(x) = 3 - |x - 3|$ on [0,6]. (4 Marks)

QUESTION THREE (20 MARKS)

a) Define Taylor's and Maclaurins series generated by a function f. (4 Marks)

b) i) Find the Taylor series generated by $f(x) = \dfrac{1}{x}$ at $a = 2$. (4 Marks)

   ii) Show that the series is geometric and converges to $\dfrac{1}{x}$. (4 Marks)

c) the total resistance R of two resistors connected in parallel is given by
   $^1/_R = {}^1/_{R_1} + \dfrac{1}{R_2}$. Approximate the change in R if R_1 increased from 10 ohms to
   10.5 ohms and R_2 decreased from 15 ohms to 13 ohms. (4 Marks)

d) Show that the point $\left(2, \dfrac{\pi}{2}\right)$ lies on the curve $r = 2\cos 2\theta$. (4 Marks)

QUESTION FOUR (20 MARKS)

a) i) find the volume of the prism whose base is the triangle in the XY-plane bounded
   by the $x - axis$ and the lines $y = x$ and $x = 1$ and whose top lies in the plane
   $f(x, y) = 3 - x - y$. (5 Marks)

   ii) Evaluate the double integral $\int_0^\pi \int_0^{\frac{\pi}{2}} \sin x^2 \cos y^2 \, dy dx$ (5 Marks)

b) i) Define an improper integral. (2 Marks)

   ii) Evaluate the integral $\int_0^\infty x e^{-2x} \, dx$ (5 Marks)

c) Find the centre of gravity of a solid hemisphere of radius r. (3 Marks)