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University Examinations 2012/2013

SECOND YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2221: CLASSICAL MECHANICS

DATE: DECEMBER 2012 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Find the impulse and its magnitude developed by the force $\vec{F} = 4t\hat{\imath} + (6t^2 - 2)\hat{\jmath} + 12\hat{k}$ from $t = 0$ *to* $t = 2$. (2 Marks)

b) A particle moves in a force field given by
$\vec{F} = (y^2 - 2xyz^3)\hat{\imath} + (3 + 2xy - x^2z^3)\hat{\jmath} + (6z - 3x^2yz^2)\hat{k}$

i. Prove that $\vec{F}$ is a conservative force field. (3 Marks)

ii. Find the potential associated with the force field. (4 Marks)

iii. Find the work done in moving the particle form (-2,-1,-2) to (-1, 3,-2) by this force field. (2 Marks)

c) A particle of mass m slides without falling down a frictionless plane AB that forms an angle $\propto$ with the horizontal. If the particle starts from rest at the top end of the incline, find the acceleration, velocity and distance travelled by the particle at any time *t*. (6 marks)

d) A particle of mass m moves along a space curve C of a force field.
$\vec{F} = (6t - 8)\hat{\imath} - 60t^3\hat{\jmath} + (20t^3 + 36t^2)\hat{k}$. Its initial position and velocity are $\overrightarrow{r_0} = 2\hat{\imath} - 3\hat{k}$ and $\overrightarrow{v_0} = 5\hat{\imath} + 4\hat{\jmath}$ respectively. Find

i. Acceleration, velocity and position of the particle at time t. (4 Marks)

ii. The momentum of the particle at a time $t = 2$. (3 Marks)

iii. Power applied to the particle at any time t. (3 Marks)

iv. Work done by the force field in moving the particle from P_1 at $t = 0$ to P_2 at $t = 2$. (3 Marks)

QUESTION TWO (20 MARKS)

A projectile is launched from the top of an inclined plane at an angle β with the horizontal. If the inclined makes an angle $\propto$ with the horizontal and also given that the initial velocity of the projectile is $\overrightarrow{v_0}$ and air resistance is negligible, show that;

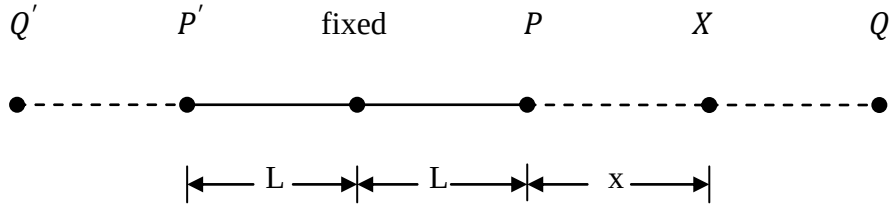
a) The range down the plane is
$R = \frac{2v_0^2 \sin \propto \cos(\propto - \beta)}{g \cos^2 \beta}$ (13 Marks)

b) The maximum range is
$\frac{v_0^2}{g(1-\sin \beta)}$ (4 Marks)

c) At what angle of launch is this achieved? (3 Marks)

QUESTION THREE (20 MARKS)

a) The figure below shows motion of a particle attached to one end of an elastic string.



Given that the length of the string is L and its mass is negligible, F_R is the restoring force due to tension and λ is the modulus of elasticity of the string at any time, show that the normal time and the time for simple harmonic motion is $T^1 = \frac{4L}{x_0}\frac{Lm}{\lambda}$ and $T = 2\pi\sqrt{\frac{Lm}{\lambda}}$ respectively. (14 Marks)

b) One end of an elastic spring is fixed to a point O on a smooth horizontal table and a particle of mass m is attached to the other end A, which is stretched to a point B. if L is the natural length of the spring an its stretched within the natural limits, show that the time of a complete oscillation of the particle is;
$T = 2\sqrt{\frac{Lm}{\lambda}}\ (\pi + \frac{2L}{\propto})$ where λ is
the elasticity of the spring. (6 Marks)

QUESTION FOUR (20 MARKS)

a) At a time $t = 0$ a parachutist having weight $w = mg$ is located at $Z_0 = 0$ and is travelling vertically downwards. If the initial velocity of the parachute is V_0 and the air resistance acting on the parachute is proportional to its speed, find;
   i. The speed and distance travelled by the parachutists at any time t where $t > 0$. (12 Marks)
   ii. The acceleration of the parachutist at any time t where $t > 0$. (4 Marks)

b) The position vectors of two particles are given as
$\overrightarrow{r_1} = t\hat{\imath} - t^2\hat{\jmath} + (2t+3)\hat{k}$ *and* $\overrightarrow{r_2} = (2t - 3t^2)\hat{\imath} + 4t\hat{\jmath} - t^3\,\hat{k}$.
Find the relative velocity and acceleration of the first particle with respect to the second one at $t = 2$. (4 Marks)