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University Examinations 2012/2013

SECOND YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND THIRD YEAR, FIRST SEMESTER FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2321: NUMERICAL ANALYSIS 1

DATE: DECEMBER 2012 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) i) Determine second Taylor polynomial for $f(x) = \cos x$ about $x_0 = 0$ and use this polynomial to approximate $\cos(0.01)$. Find also the error bound if this approximation is used for the value of $\cos(0.01)$. (6 Marks)

b) ii) Suppose p^* must approximate p with relative error of at most 10^{-3}. Find the largest interval in which p^* must lie if P=150. (3 Marks)

c) i) Let $f(x) = -x^3 - \cos x$ and $x_0 = -1$. Use Newton – Raphson's method to find x_3. (4 Marks)

ii) Divide $3x^3 - 2x^2 + 3x - 4$ by $x - 3$. Using synthetic division. Write answer in the form.$(q(x) + \frac{r(x)}{d(x)}$. (5 Marks)

d) Applying Newton's forward interpolation formula, compute the value of $\sqrt{5.5}$ given that $\sqrt{5} = 2.236, \sqrt{6} = 2.449, \sqrt{7} = 2.646$ *and* $\sqrt{8} = 2.828$ correct to 3 places of decimal. (6 Marks)

QUESTION TWO (20 MARKS)

a) i) Derive Newton – Raphson method of finding the zeros of polynomials.(6 Marks)

ii) State the intermediate value theorem. (2 Marks)

iii) Show that the equation $x \cos x - 2x^2 + 3x + 1 = 0$ has at least one root between $[0.2, 0.3]$. (2 Marks)

b) Using Newton-Raphson method, evaluate to 3.dp

i. $\sqrt{12}$ (3 Marks)

ii. $\sqrt[3]{17}$ (3 Marks)

c) Use the Bisection method upto the 3rd interation to find the root of $\sqrt{x} - \cos x = 0$ on $[0,1]$. (4 Marks)

QUESTION THREE (20 MARKS)

a) The table below gives values of a function at various points.

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

Find the approximation to $f(1.5)$ using

i. Second – degree lagrange polynomial (7 Marks)

ii. Fourth – degree lagrange polynomial (7 Marks)

b) i) State the approximate interpolation formula which is to be used to calculate the value of $e^{1.75}$ from the following data and hence evaluate it from the given data.

x	1.7	1.8	1.9	2.0
e^x	5.474	6.050	6.686	7.389

(6 Marks)

QUESTION FOUR (20 MARKS)

a) values for $f(x) = \ln x$ are given in the table

x	$f(x)$
2.0	0.69315
2.2	0.78846
2.6	0.95551

Find the approximate value of;

i. $f'(2.0)$ using linear interpolation. (4 Marks)

ii. $f"\ (2.0)$ using quadratic interpolation. (6 Marks)

b) Using the forward difference formula find the first and second derivatives of $x^{1/2}$ at $x = 15$ from the table below

x	15	17	19	21	23	25
$x^{1/2}$	3.873	4.123	4.359	4.583	4.796	5.000

QUESTION FIVE (20 MARKS)

i. Using the general quadrature formula derive Simpson's one third rule. (6 Marks)

ii. Using Simpson's one – third rule find

$$\int_0^6 \frac{dx}{(1+x)^2}$$

(4 Marks)

iii. A river is 80m wide. The depth y of the river at a distance 'x' from one bank is given by the following table

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the approximate area of cross-section of the river using Simpson's $3/8$ rule.

(5 Marks)

iv. Using Euler – Maclaurin formula evaluate

(5 Marks)

$$\int_0^1 \frac{dx}{1+x^2}$$