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University Examinations 2013/2014

SECOND YEAR, SECOND SEMESTER EXAMINATIONS FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2220: VECTOR ANALYSIS

DATE: DECEMBER 2013 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) Three forces of $4N, 6N$ *and* $8N$ act on a point as show in Fig. 1. Calculate the magnitude of the resultant force and its direction relative to the 4N force. (4 Marks)

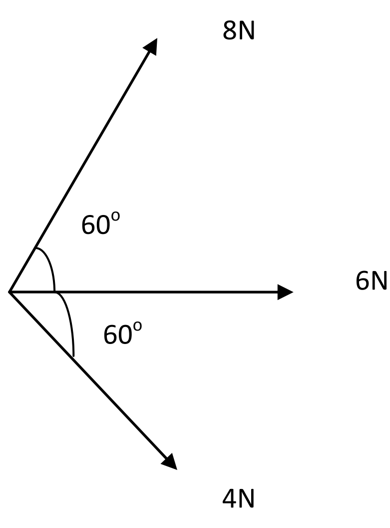

Fig. 1

(b) Given that $\vec{r}(t) = t\sin t\,\hat{i} + t\cos t\,\hat{j} + t^2\,\hat{k}$, compute $\dfrac{d\vec{r}}{dt}$ at $t = \dfrac{\pi}{2}$. (3 Marks)

(c) A surface has an equation given by $x^3 + 3xy + z^2 = 11$. Find the equation of the tangent plane to this surface at the point $(1, 2, 2)$. (4 Marks)

(d) Find the parametric equations and the rectangular equations for the line through the points $P(3,2,1)$ and $Q(-1,2,4)$. (4 Marks)

(e) Given that $\vec{a} = \hat{i} - 3\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}$, find the angle between vectors $\vec{a} + \vec{b}$ and $\vec{b} - 2\vec{c}$. (4 Marks)

(f) The Cartesian equation of a cone is given by $x^2 + y^2 = z^2$. Find an equation for this cone in spherical co-ordinates (simplify your answer). (3 Marks)

(g) Find div $\vec{F}$ at the point $(3,2,1)$ given that $\vec{F}(x,y,z) = e^x \sin y\,\hat{i} - e^x \cos y\,\hat{j} + z^2\hat{k}$.
(4 Marks)

(h) Evaluate $\int_c y\,dx + x^2 dy$ where c is the parabolic arc given by $y = 4x - x^2$ from $(5,0)$ to $(2,3)$ (4 Marks)

QUESTION TWO – (20 MARKS)

(a) Given that $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

i. Find a unit vector normal to the plane containing vectors $\vec{a} + (\vec{a}.\vec{b})\vec{c}$ and $\vec{c}$.
(5 Marks)

ii. Show that the normal to the plane containing the vectors $\vec{a}$ and $\vec{b}$ and the normal to a plane containing vectors $(\vec{b}.\vec{c})\vec{b}$ are parallel. (5 Marks)

(b) The vector $\vec{r}(t) = 3\cos t\hat{i} + 3\sin t\,\hat{j} + (t^3 - t^2)\hat{k}$ gives the position of a moving body at time t. Find the times at which the body's velocity and acceleration are orthogonal.
(5 Marks)

(c) Compute the values of constant a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has a maximum of magnitude 64 in a direction parallel to the z-axis. (5 Marks)

QUESTION THREE – (20 MARKS)

(a) (i) State Green's theorem in the plane. (2 Marks)

(ii) Verify Green's theorem in the plane for $\oint_c (xy - x^2)dx + x^2\ y\,dy$ where c is a triangle $O(0,0)$, $A(1,0)$, $B(1,1)$ (10 Marks)

(b) Given that $\vec{F}(x,y,z)=\hat{i}+3x\hat{j}+2y\hat{k}, \quad \vec{G}(x,y,z)=x\hat{i}-y\hat{j}+z\hat{k}$, find $\vec{\nabla}\bullet\left(\vec{F}\times\vec{G}\right)$

(5 Marks)

(c) Use vector method to show that the points $P(2,-1,5)$, $Q(6,0,6)$, $R(14,2,8)$ are collinear.

(3 Marks)

QUESTION FOUR – (20 MARKS)

(a) A space curve is represented by the vector equation $\vec{r}(t)=e^t\cos t\hat{i}+e^t\sin t\hat{j}+e^t\hat{k}$.

Compute (simplifying your answers):

i. The unit tangent vector, $\vec{T}$ of the curve. (4 Marks)

ii. The principal normal, $\vec{N}$ of the curve (3 Marks)

iii. The Binomial vector, $\vec{B}$ (3 Marks)

(b) Convert the point $\left(3,\frac{\pi}{4},1\right)$ from cylindrical coordinates to rectangular coordinates.

(2 Marks)

(c) Given that $\vec{A}=2\hat{i}-3\hat{j}+6\hat{k}$ and $\vec{B}=2\hat{i}+4\hat{j}+4\hat{k}$, find $proj_{\vec{A}}\vec{B}$ (3 Marks)

(d) Using the curl test, determine whether

$\vec{F}(x,y,z)=\left(3x^2y^2+yz^2\right)\hat{i}+\left(2x^3y+xz^2\right)\hat{j}+2xyz\hat{k}$ is conservative and if it is, find a scalar function associated with $\vec{F}(x,y,z)$ such that $\vec{\nabla}\phi=\vec{F}(x,y,z)$ (5 Marks)