MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY

P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2012/2013

SECOND YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2220: VECTOR ANALYSIS

DATE: DECEMBER 2012 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Verify that the following points are not collinear hence find an equation of the plane containing these points $A(3,5,-2), B(5,2,1), C(7,7,4)$. (6 Marks)

b) Using vectors $\vec{A} =< 2,1,4 > and\ B =< 3,5,-1 >$.

  i. Compute $\vec{A}.\vec{B}$. (2 Marks)

  ii. Find the angle between the vectors $\vec{A}$ and $\vec{B}$. (3 Marks)

c) Compute $div(\vec{B} \times \vec{A})$ given that $\vec{A} = 2xy\boldsymbol{i} + x^2z\boldsymbol{j} + \boldsymbol{k}$ and $\vec{B} = yz\boldsymbol{i} + xy\boldsymbol{j} - x\boldsymbol{k}$**.** (3 Marks)

d) Convert the point $(1,\sqrt{2}, 1)$ from Cartesian to spherical coordinates. (5 Marks)

e) Compute the arc length of the curve $\vec{r}(t) = 3\sin 4t\,\boldsymbol{i} + 3\cos 4t\,\boldsymbol{j} + 5t\boldsymbol{k}$ in the interval $0 \le t \le 2\pi$ giving your answer in terms of π. (4 Marks)

f) A plane has a normal vector $\boldsymbol{n} < 3,-2,-4 >$. Find the equation of this plane if the point (1,3,2) lies on this plane. (3 Marks)

g) Find the volume of the parallelpiped bound by the vectors $\vec{A} =< 2,-1,4 >$, $\vec{B} =< 1,3,-2 > and\ \vec{C} =< 3,1,1 >$ where 1 unit is 1 meter. (4 Marks)

QUESTION TWO (20 MARKS)

a) Given the points $A(-1,3,\sqrt{3})\ and\ B(4,6,0)$

  i. Find the angle that the vector $\overrightarrow{AB}$ makes with each of the coordinates axis. (5 Marks)

  ii. Compute $proj_{\vec{A}}\vec{B}$ where $\vec{A} = \overrightarrow{OA}$ and $\vec{B} = \overrightarrow{OB}$. (4 Marks)

b) Given an ellipsoid $x^2 + 3y^2 + 4z^2 = 25$, compute the following at the point P(3,2,1)

  i. The parametric equation of the normal line. (5 Marks)

ii. The equation of the tangent plane to the ellipsoid. (3 Marks)

c) Compute the unit tangent vector to the curve $\vec{r}(t) = 2\sin 3t\,\boldsymbol{i} + t\boldsymbol{j} - 2\cos 3t\,\boldsymbol{k}$. (3 Marks)

QUESTION THREE (20 MARKS)

a) Find the directional derivative to the function $f(x,y) = x^2 + 4xy$ at the point $(1,2)$ in the direction from $A(-2,1) to\ B(3,2)$. (5 Marks)

b) Given the points $A(1,-3,6), B(9,-9,1) and\ C(-6,-29,26)$ in space.

i. Find the lengths of a triangle ABC. (6 Marks)

ii. Prove that triangle ABC is right angled and state the vertex at which it is right angled. (3 Marks)

c) If $\vec{f} = (3x^2 + 6y)\boldsymbol{i} - 14yz\boldsymbol{j} + 20xz^2\boldsymbol{k}$ find the work done in moving the force field $\vec{f}$ along a curve C given by $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1).(6 Marks)

QUESTION FOUR (20 MARKS)

a) The position of a particle in space at time t is given by $\vec{S} = 2\cos 2t\,\boldsymbol{i} + 2\sin 2t\,\boldsymbol{j} + 3t\boldsymbol{k}$.

i. Find the velocity and acceleration of the particle at any time. (3 Marks)

ii. Find the speed of the particle. (3 Marks)

iii. Find the length along the curve of the trajectory from the point A(2,0,0) to B(2,0,3π). (4 Marks)

b) Use the divergence theorem to evaluate the integral $\iint_s \vec{F}.\vec{n}ds$ where $\vec{F} = 2x^2y\boldsymbol{i} + yz\boldsymbol{j} - z^2\boldsymbol{k}$ and S is the surface area of the cube bounded by $x = 1, x = 2, y = 1, y = 2, z = 1, z = 2$. (10 Marks)