EGERTON UNIVERSITY

MATHEMATICS DEPARTMENT

MATH 210 -- C.A.T 2

1. Given a matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$.

   a) Find the characteristic and minimal polynomials of A. $\lambda^3-\lambda^2-6\lambda-31$

   b) Show that A is a zero of its own characteristic polynomial.

   c) Determine eigenspaces corresponding to each eigenvalue.

   d) Give the geometric and algebraic multiplicities of each eigenvalue of A.

   e) Is matrix A diagonalizable? If so, use (c) to find the diagonal matrix of A.

2. a) Given that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a map defined by
   $T(\underset{\sim}{x}) = T(x_1,x_2) = (x_1 + 2x_2,\ x_1)$. Is T as defined linear?

   b) Given that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} 2u_1 - 3u_2 + u_3 \\ u_1 - 2u_2 + u_3 \\ u_1 - 3u_2 + 2u_3 \end{bmatrix}$$

   (i) Find the matrix of T w.r.t. the standard basis of $\mathbb{R}^3$.

   (ii) Use the similarity theorem to compute the matrix of T w.r.t the basis B' if
   $B' = \{v_1 = (1,1,1),\ v_2 = (1,0,2),\ v_3 = (0,1,0)\}$.

3. a) If $\underset{\sim}{u} = (1,-2,0,1)$ *and* $\underset{\sim}{v} = (-3,1,0,-2)$ are vectors in an inner product space $\mathbb{R}^4$ with Euclidean inner product, find the angle θ between them.

   b) Given an inner product $<\underset{\sim}{u},\underset{\sim}{v}> = 2u_1v_1 + u_2v_2 + u_3v_3$ for vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ in $\mathbb{R}^3$.

   (I) Is this an inner product space?

   (ii) Transform the vectors $\underset{\sim}{u}_1 = (1,1,1)$, $\underset{\sim}{u}_2 = (1,1,0)$ *and* $\underset{\sim}{u}_3 = (1,0,0)$ into an orthonormal basis.

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