# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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**University Examinations 2013/2014**

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR MASTER OF SCIENCE IN STATISTICS

**STA 3113: DECISION THEORY**

**DATE: APRIL 2014** **TIME: 3 HOURS**

**INSTRUCTIONS:** *Answer question **one** and any other **two** questions.*

**QUESTION ONE – (30 MARKS)**

(a) Define the following terminologies as used in Bayesian statistics (Hint: Use formula where applicable).
   (i) Prior distribution
   (ii) Posterior distribution
   (iii) Risk function
   (iv) Bayes action (4 Marks)

(b) Consider an experiment involving two bags. One bag contains 7 black and 2 white balls. The second bag contains 6 black and 5 white balls. By flipping a coin, we pick a ball from the first bag if head is obtained and second bag otherwise. What is the conditional probability of picking a ball from bag 1 given that the ball selected is white? (4 Marks)

(c) Consider a random sample $X = (x_1, x_2, ..., x_n)$ where x is Bernouli distributed with probability of success p. Obtain
   (i) The likelihood function for the random sample. (2 Marks)
   (ii) The posterior distribution of p if the prior density of p is assumed to be Beta$(\propto, \beta)$ (6 Marks)

(iii) Express the posterior mean as a sum of the prior mean and the $MLE(p)$.
(2 Marks)

(iv) Is the Beta prior applied on a Bernouli likelihood a conjugate prior?
(2 Marks)

(d) Let $X = (x_1, x_2, ..., x_n)$ be a random sample from a Binomial distribution $(K, \theta)$. Suppose the prior distribution of $\theta$ is $\pi(\theta) \sim Beta(\propto, \beta)$. Let the loss function for Bayesian decision making be $L(\theta, a) = a^2 - 2\theta a + a^2$. Determine

(i) The prior density of $\theta$ (5 Marks)

(ii) The Bayes action for the loss function defined above. (Hint: Obtain the Posterior mean). (5 Marks)

**QUESTION TWO – (20 MARKS)**

Consider a random sample $X = (x_1, x_2, ..., x_n)$ from a normal population with mean $\mu$ and unknown precision $\tau$.
$X \sim N(\mu, \tau)$ .

(a) Suppose $\tau$ is assumed to have a Gamma $(\propto, \beta)$ prior distribution. Show that $\tau$ has a Posterior Gamma density function given by the hyper parameters

$$\alpha^* = n/2 + \alpha$$

(12 Marks)

$$\beta^* = \left[ \frac{1}{2} \sum_{i=1}^{n} (x_i - \mu)^2 + \frac{1}{\beta} \right]^{-1}$$

(b) Obtain the posterior mean under the quadratic loss function. (8 Marks)

**QUESTION THREE – (20 MARKS)**

(a) Consider a random sample $X = (x_1, x_2, ..., x_n)$ from a Poisson $(\lambda)$ distribution. Assume$\lambda \sim Gamma\ (\propto, \beta)$. Using the los function $L(\hat{\lambda}, \lambda) = (\lambda - \hat{\lambda})^2$, show that the Posterior mean is given by $\hat{\lambda} = \frac{n}{n+\beta} \bar{x} + \frac{\beta}{n+\beta} \bullet \frac{\alpha}{\beta}$ (10 Marks)

(b) (i) Consider a random sample $X = (x_1, x_2, ..., x_n)$ from a Normal $(\theta, 1)$ distribution. Suppose the prior density of $\theta$ is $p(\theta) = 1;\ -\infty < \theta < \infty$ show that the $(1-\propto)100\%$ HPD credible set for $\theta$ is given by $\bar{x} \pm z_{\alpha/2} \sqrt{\frac{1}{n}}$ (7 Marks)

(ii) Hence for a random sample size $n = 25$ and $\sum_{i=1}^{25} x_i = 1250$, obtain the 95% HPD credible set for the parameter $\theta$. (3 Marks)

**QUESTION FOUR – (20 MARKS)**

(a) $X \sim N(\theta, \sigma^2)$

(i) Show that the the $MLE(\theta) = \bar{x}$ (3 Marks)

(ii) Is $\hat{\theta} = MLE(\theta)$ unbiased for $\theta$? (2 Marks)

(iii) Obtain the Fisher's information $I(\theta)$ (2 Marks)

(iv) Obtainer the Crammer Rao Lower bound for the variance of the estimator of $\theta$ (2 Marks)

(v) Determine the Jeffrey's prior for $\theta$ and comment on the suitability of such a prior density. (3 Marks)

(b) Use the Jeffrey's prior obtained above to obtain the Posterior density for $\theta$. (8 Marks)

**QUESTION FIVE – (20 MARKS)**

(a) Let $X \sim Poisson\ (\lambda)$. Choose an improper prior of the form.

$p(\lambda) = C;\ -\infty < \lambda < \infty$

(i) Show that the Posterior mode is equal to the maximum likelihood estimate for $\lambda$.

(ii) Obtain the Posterior mode. (10 Marks)

(b) For $X \sim Poisson\ (\lambda)$, obtain the Jeffrey's prior for $\lambda$ and use it to obtain the Posterior mean. (10 Marks)