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University Examinations 2013/2014

FIRSTYEAR, FIRST SEMESTER EXAMINATIONS FOR MASTER OF SCIENCE IN STATISTICS

## STA 3104: DESIGN AND ANALYSIS OF EXPERIMENTS

INSTRUCTIONS: Answer question one and any other two questions.
QUESTION ONE - (30 MARKS)
(a) Define the following terminologies (if possible, with illustration)
(i) Latin Square Design
(ii) Greaco-Latin Square Design
(iii) D-Optimality and A-Optimality of design
(6 Marks)
(b) The following data is not a perfect Latin Square design data

| A | C | B | D |
| :--- | :--- | :--- | :--- |
| 10 | 15 | 5 | 4 |
| C | B | A | D |
| 12 | 10 | 3 | 4 |
| B | D | A | C |
| 20 | 10 | 5 | 10 |
| D | A | B | C |
| 10 | 4 | 12 | 25 |

(i) Construct a perfect Latin Square design.
(2 Marks)
(ii) Compute the following sum of squares for the Latin square obtained in (i).

SS(treatment), SS(rows), SS(columns)
(8 Marks)
(c) (i) Complete the ANOVA Table below for a completely Randomized Block Design (CRBD).

| Source of <br> variation | df | SS | MSS | F |
| :--- | :--- | :--- | :--- | :--- |
| Treatments | 3 | 21.7 | 7.23 | - |
| Blocks | 2 | 32.2 | - | - |
| Errors | - | - | - |  |
| total | 11 | 57.7 |  |  |

(ii) Test at $\propto=0.05$ if the treatments and blocks are statistically significant.
(4 Marks)
(iii) Compute the relative efficiency of the RCBD above over a CRD for the same experiment. What is the gain in efficiency?
(4 Marks)

## QUESTION TWO - (20 MARKS)

(a) Consider the Randomized Complete Bloc Design (RCBD). For a fixed effects model, with V-treatments and B-blocks, and $\sum_{i=1}^{v} t_{i}=\sum_{j=1}^{b} B_{j}=0$ where $t_{i}=$ treatment effects and $B_{j}=$ block effects, show that
(i) $E(\operatorname{MSS}($ treatments $))=\sigma_{e}{ }^{2}+\frac{b}{v-1} \sum_{i=1}^{v} t_{i}{ }^{2}$
(ii) $E(\operatorname{MSS}($ Blocks $))=\sigma_{e}{ }^{2}+\frac{v}{b-1} \sum_{j=1}^{b} b_{j}{ }^{2}$
(5 Marks)
(iii)Under $H_{0}: t_{i}=0 ; B_{j}=0$ comment if $M S S(t)$ and $M S S(B)$ are unbiased for the error variance.
(2 Marks)
(b) The data below is an experiment in which four treatments are laid out in 3 blocks.

Block

| Treatments |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 |
| 1 | 6 | 8 | 7 | 11 |
| 2 | 5 | 6 | 6 | 9 |
| 3 | 9 | 9 | 11 | 12 |

At $\propto=0.01$, test the hypothesis $H_{0}: t_{1}=t_{2}=t_{3}=t_{4}=0 ; B_{1}=B_{2}=B_{3}=0$
Versus $H_{1}$ : at least one treatment or block effect differs from zero.
(8 Marks)

## QUESTION THREE - (20 MARKS)

Four fertilizers are applied to a farm, each with five replications. The following yield of maize in bags is recorded.

| A | 55 | 49 | 42 | 21 | 52 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B | 61 | 112 | 30 | 89 | 63 |
| C | 42 | 97 | 81 | 95 | 92 |
| D | 169 | 137 | 169 | 85 | 154 |

(a) Identify the type of design used in this experiment.
(2 Marks)
(b) Derive the formulae for computation of sums of squares necessary for the ANOVA analysis. Hint: split the formula for total sums of squares.
(c) At $\alpha=0.01$, test the hypothesis
$H_{0}: t_{A}=t_{B}=t_{C}=t_{D}=0$ versus
$H_{1}$ : at least one effect differs from zero.
(d) For any two pairs of means and a balance design, we reject

$$
H_{0}: t_{i}=t_{i}^{\prime} \text { for } H_{i}: t_{i} \neq t_{i}^{\prime} \text { if } \frac{\hat{t}_{i}-\hat{t}_{i}^{\prime}}{\hat{\sigma}_{e} \sqrt{\frac{1}{n_{i}}+\frac{1}{n_{i}^{\prime}}}}>t_{(\alpha / 2, d f=n-v)}
$$

Based on this judgment, compare all the possible pairs of means.
(7 Marks)
(e) Recommend one fertilizer to the farmers.
(1 Mark)

## QUESTION FOUR - (20 MARKS)

(a) Suppose the Yij-th observation in the jth block receiving treatment i is missing, derive the formula for estimating the missing observation in a Randomized Complete Block Design. $(i=1,2, \ldots v ; j=1,2, \ldots, b)$
(10 Marks)
(b) Hence set up an ANOVA at $\alpha=0.05$ for the cow feeds data below.

HERDS
COW FEDS

| HERDS |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 | 3 | 4 |
| A | 19 | - | 23 | 26 |
| B | 26 | 28 | 27 | 33 |
| C | 20 | 29 | 22 | 26 |

Note that sums of squares due to treatment is over-estimated by a factor

$$
p=\frac{\left(B_{J}{ }^{\prime}+V T_{i}{ }^{\prime}-G\right)^{2}}{V(V-1)(b-1)^{2}}
$$

