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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR DEGREE OF MASTER OF SCIENCE IN STATISTICS

STA 3101: MULTIVARIATE ANALYSIS

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions.*

QUESTION ONE – (30 MARKS)

(a) Let $\underset{-}{y}$ be a $p \times 1$ random vector and $\underset{-}{x}$ be also a $p \times 1$ matrix of constants. Show that $Cov(X,Y) = \Sigma_{X,Y}$ Let A be a $l \times p$ matrix of constants at B be a $p \times q$ matrix of constants, show that $Cov(AY, BX) = A\sum_{YX} B^{/}$ (8 Marks)

(b)

(i) Define the Hotteling T^2 – Statistics

(ii) Show that the statistic T^2 is invariant of non-singular linear transformation.

(c) Let $\underset{-}{x}$ be a $p \times 1$ random vector with $V(\underset{-}{x}) = \sum$ *and* $E(\underset{-}{x}) = \underset{-}{\mu}$. Show that

(i) The matrix Σ is symmetric positive definite if the component random variables $\underset{-}{x}$ are linearly independent. (8 Marks)

(ii) For a quadratic form

$$Q = (\underset{-}{x}) = \underset{-}{x}^{/} \, A \, \underset{-}{X}$$

$$E\left[Q\left(\underset{-}{X}\right)\right] = trace\,\left(A\sum\right) + \underset{-}{\mu}^{/} \, A \, \underset{-}{\mu}$$

If A is symmetric positive definite matrix. (6 Marks)

QUESTION TWO – (20 MARKS)

(a) Show that $\underset{-}{x}$ is $N_P(\mu, \Sigma)$ iff $a^{/}x$ is $N_1\left(a^{/}\mu, \underset{-}{a}^{/} \Sigma \underset{-}{a}\right)$ (6 Marks)

(b) (i) Briefly explain the importance of principal component analysis statistics.
(4 Marks)

(iii) Let $\underset{-}{x}_1 \sim N\left(\underset{-}{\mu}_1 \,, \Sigma\right)$ and $\underset{-}{x}_2 \sim N\left(\underset{-}{\mu}_2, \Sigma\right)$ denote random vectors with corresponding multivariate normal distribution. Assume $\underset{-}{x}_1$ and $\underset{-}{x}_2$ are independents. Assuming Σ is unknown consider testing

$$H_0 = \underset{-}{\mu}_1 = \underset{-}{\mu}_2$$

vs . Determine a test for the hypothesis basing on the principal

$$H_1 = \underset{-1}{\mu} \neq \mu_2$$

component of $\underset{-}{x}_1$ and $\underset{-}{x}_2$

QUESTION THREE – (20 MARKS)

(a) $f\left(\underset{-}{x}\right) = \dfrac{1}{(2\pi)^{\frac{p}{2}} |\Sigma|^{\frac{1}{2}}} \exp - \dfrac{1}{2}\left(\underset{-}{x} - \underset{-}{\mu}\right)^{1} \Sigma^{-1}\left(\underset{-}{x} - \underset{-}{\mu}\right)$ where $\underset{-}{x}$ is $p \times 1$ vector and Σ is a positive definite matrix. Show that

(i) $f\left(\underset{-}{x}\right) \geq 0$

(ii) $\int_{-\infty}^{\infty} f\left(\underset{-}{x}\right) d\,\underset{-}{x} = 1$

Where the integral denotes the p definite multiple integrals each over the interval $(-\infty, \infty)$.

(8 Marks)

(b) Let y be distributed as $N_P(\mu, \Sigma)$. Show that the moment generating function of $\underset{-}{y}$ is

$M_y(t) = \exp \underset{-}{t}' \underset{-}{\mu} + t' \frac{\Sigma}{2} \underset{-}{t}$, where $\Sigma = Var\left(\underset{-}{y}\right)$ (6 Marks)

(c) Using part (b) or otherwise, show that the moment generating function of $\underset{-}{Z} = \underset{-}{y} - \underset{-}{M}$ is

$M_2(t) = \exp \underset{-}{t}' \frac{\Sigma}{2} \underset{-}{t}$.

QUESTION FOUR –(20MARKS)

(a) Show that if $\underset{-}{y}$ is $N_P(\mu, \Sigma)$then an $r \times 1$ subvector of $\underset{-}{y}$ has an r-variate normal distribution with the same means, variance and covariances as in the original p-variate normal distribution. (6 Marks)

(b) Using part (a) or otherwise, show any individual variable y_i in $\underset{-}{y}$ is distributed as $N_1(\mu_i, \delta_{ii})$ where $\mu_i = E(y_i)$ and $\delta_{ii} = var\ (y_i)$ (6 Marks)

(c) Show that if $\underset{-}{y}$ and $\underset{-}{x}$ are jointly multivariate normal with $\sum_{ix} \neq 0,$ then the conditional distribution of $\underset{-}{y}$ given $\underset{-}{x}$, $f\left(\underset{-}{y}\middle|\underset{-}{x}\right)$ is multivariate normal with mean and covariance matrix E

$$E\left(\underset{-}{y}\middle|\underset{-}{x}\right) = \mu y + \Sigma_{yx} \Sigma_{xx}^{-1}\left(\underset{-}{x} - \underset{-}{\mu}\ \ x\right)$$

$$\text{var}\left(\underset{-}{y}\middle|\underset{-}{x}\right) = \Sigma_{yy} - \Sigma_{yx} \Sigma_{xx}^{-1} \Sigma_{yx}$$

respectively. (8 Marks)

QUESTION FIVE – (20 MARKS)

Suppose that the random vector $\underset{\sim}{y}$ is distributed as $N_4[\mu, \Sigma]$ where $\mu = (1\ 2\ 3\ -2)'$ and

$$\Sigma = \begin{bmatrix} 4 & 2 & -1 & 2 \\ 2 & 6 & 3 & -2 \\ -1 & 3 & 5 & -4 \\ 2 & -2 & -4 & 4 \end{bmatrix}$$

(a) Find the distribution of

(i) $(y_1\ y_2)'$

(ii) y_2

(iii) $z = y_1 + 2y_2 - y_3 + 3y_4$

(iv) $\underset{\sim}{z} = (z_1, z_2)'$, where $z_1 = y_1 + y_2 - y_3 - y_4$ *and* $z_2 = -3y_1 + y_2 + 2y_3 - 2y_4$

(v) $(y_1, y_2)'$ given y_3, y_4 (12 Marks)

(b) Find

(i) $Corr(y_1, y_3)$

(ii) $Corr(y_1 y_3 | y_3 y_4)$ (8 Marks)