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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3107: NON PARAMETRIC REGRESSION

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) Let $Y \in (0,1)$ denote the outcome of a coin toss with $\pi = p(y=1)$ and $1-\pi = p(y=0)$.

Define $\log it(\pi_i) = \beta_0 + \sum_{j=1}^{p} \beta_j x_{ij} = x_i^{/}\beta$.

(i) Show that the MLE of π is $\sum y_i / n$

(ii) Write down an algorithm to be used to estimate the parameters β_0, β_j.

(b) (i) What is an exponential family

(ii) Show that $y \sim Poisson\ (\theta)$ and $y \sim Binomial$ belongs to exponential family.

(c)

(i) Define what is meant by a spline?

(ii) Illustrate (i) by two examples.

(iii) Solve the problem

$\arg\min_{\beta} (Y - B\beta)^{/}(Y - B\beta) + \lambda \beta^{/} \Omega \beta$, where B- represents a spline.

(iv) Show that splines are linear smoothers.

QUESTION TWO – (20 MARKS)

(a) Let $\hat{r}_u(x)$ be a linear smoother show that $\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n}(y_i - \hat{r}(x_i))^2}{n - 2r + \tilde{r}}$

(b) Given the data $(x_1, y_1), - - (x_n, y_n)$. Explain how you will find a non parametric relationship between $y's$ and $x's$

QUESTION THREE – (20 MARKS)

Suppose that $x's \in [a, b]$ explain how you will obtain the relationship $\hat{y} = \hat{r}_u(x)$ using regressogram procedure

(a) Obtain the bias of the estimate $if\,[a, b] = [0,1]$

(b) Obtain the variance of the estimator $if\,[a, b] = [0,1]$

(c) What is your conclusion in (a) and (b) above.

QUESTION FOUR – (20 MARKS)

(a) What is a Kernel regression. Illustrate your explanation with examples.

(b) To obtain a smoothing matrix one could minimize $Gcv(h) = \dfrac{1}{n}\sum_{i=1}^{n}\left(\dfrac{y_i - \hat{r}_u(x)}{1 - r/n}\right)^2$ where

$r = tr(L)$ is the effective degree of freedom $G\ C\ V(h) \approx \left(1 + \dfrac{2r}{n}\right)\hat{\sigma}^2$

(c) Give a polynomial $p_x(u; a) = a_0 + a_1(u - x) + a_1(u - x)^2 + ... + \dfrac{ap}{p!}(u - x)^p$. Explain how you will obtain the estimate of $a = (a_0, ...a_p)'$

QUESTION FIVE – (20 MARKS)

(a) Describe the local average procedure for non parametric method.

(b) Show that a linear regression estimator is a special case of non parametric estimator.

(c) Solve for $\underset{\beta}{\arg\min}\ (Y - X\beta)'(Y - X\beta) + \lambda\beta' I\beta$