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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3100: PROBABILITY THEORY

DATE: DECEMBER 2013 **TIME: 3HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

QUESTION ONE - (30 MARKS)

a) Let $\{A_i\}$ be a sequence of sets with relative to $\{\Omega, A, p\}$
   i. Define limit superior of $\{A_i\}$ and limit inferior of $\{A_i\}$.
   ii. Show that limit superior and inferior and elements of A.

b) Let $\{x_n\}$ be a sequence of random variable defined to the probability space $\{\Omega, A, p\}$ with converges to the random variable $X \epsilon R$ with probability 1. If $g: \mathbb{R} \rightarrow \mathbb{R}$ is continuous, then show that the sequence $\{g(x_n)\} \longrightarrow g(x)$ w.p.1

c) i) Define the term simple random variable.
   ii) If x is a random variable. Show that $|X|$is a random variable.
   iii) $x_1 - x_2$ is a random variable if both $x_1$and x_2 are random variables.
   iv) $x^2 + 2x + 1$ is a random variable.

QUESTION TWO (20 MARKS)

a) What is a probability measure P?

b) What is a probability space (Ω, A, p)?

c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a non-negative measurable function with $\int_{-\infty}^{\infty} f(x, dx = m, where\ 0 < m < \infty$ show that if E is a Borel set, define $P(E) = \frac{1}{m}\int_E\ f(x)dx,$ then P is a probability measure on $(\mathbb{R}, \mathcal{B})$

d) Let E be an event such that $P(E) \neq 0$. Show that the set function defined by $P(A|E) = P(AnE)/P(E)$ for any set A is a probability measure.

QUESTION THREE (20 MARKS)

a) If A_1 and A_2 are independent events relative to (Ω, A, p). Show that A_1^C and A_2^C are also independent event.

b) For each sequence $[A_n]$ defined relative to (Ω, A, p). Show that

$$P_r\begin{pmatrix} liminf A_n \\ n \end{pmatrix} \leq \begin{matrix} limit \\ n \end{matrix} \quad P(A_n) \leq \begin{matrix} limsup P(A_n) \\ n \end{matrix} \leq P\begin{pmatrix} limsup A_n \\ n \end{pmatrix}$$

c) Let $E_1, E_2, \ldots$. be a sequence of events $m(\Omega, A, p)$. Show that

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} P(E_n)$$

QUESTION FOUR (20 MARKS)

a) Let (Ω, A) be a measurable space and let $p_1, p_2, \ldots$ be a sequence of probability measures define as t. prove that the function p* defined on ***A*** by

$$P^*(E) = \sum_{n=1}^{\infty} \frac{1}{2^n} P_n(E)$$ is a probability measure on **A**.

b) Let (Ω, A, p) be a probability space and let x be a random variable defined over R. Define a function P' over the set $\mathcal{B}$ of borel sets of $\mathbb{R}$ by $P'(\mathcal{B}) = P\{x^{-1}(B)\} B \epsilon \mathcal{B}$. Show that the triple $(\mathbb{R}, \mathcal{B}, P')$ is a probability space

c) Define the following terms

i. The sequence $\{x_n\} \longrightarrow x$ uniformly on $A \epsilon \mathbf{A}$ of $(\Omega, \mathbf{A}, P)$

ii. $\{x_n\} \overset{p.a.e}{\rightarrow} x$

iii. $\{x_n\} \overset{p}{\rightarrow} x$

iv. $\{x_n\} \overset{w.p.1}{\rightarrow} x$

QUESTION FIVE (20 MARKS)

a) Prove that

i. $\begin{pmatrix} \lim sup A_n \\ n \end{pmatrix} \cap \begin{matrix} limsup B_n \\ n \end{matrix} \supset \begin{matrix} limsup A_n \cap B_n \\ n \end{matrix}$

ii. $\begin{pmatrix} \lim sup A_n \\ n \end{pmatrix} \cup \begin{matrix} limsup B_n \\ n \end{matrix} = \begin{matrix} limsup A_n \cup B_n \\ n \end{matrix}$

b) If X is a random variable, show that

i. $x + 2$ is a random variable

ii. $x^2 + 3x + 1$ is a random variable

c) Let $\{E_1, E_2, \ldots E_n\}$ be a collection of mutually independent events

i. Show that the probability of the occurrence of atleast one of the events is given by

$$1-\prod_{i=1}^{n}\left(1-p\left(E_i\right)\right)$$

ii. Show that the probability of the occurrence of exactly one of the events say E_1 is given by

$$p(E_i)-\prod_{k=1}^{n}\left(1-p\left(E_k\right)\right) \quad k \neq i$$