MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY
P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3100: PROBABILITY THEORY

DATE: JANUARY 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer Question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Briefly explain the meaning of the following terms/phrases as used in probability and statistical theory.
   i. Set
   ii. Power set
   iii. Class
   iv. Field
   v. Borel set
   vi. σ −field
   vii. Function
   viii. Random variable
   ix. Monotone increasing
   x. Monotone decreasing (10 Marks)

b) Show that the total number of subsets contained in its power set is 2^n where n is the number of elements in the set. (5 Marks)

c) Confirm that when given a class Ai,i=1,2,….n of n sets, there exists a class Bi,i=1,2,…n such that Bi's are disjoint and
$\bigcup_{i=1}^{n} Ai = \sum_{i=1}^{n} Bi$ (5 Marks)

d) Prove that a σ-field is a monotone field and vice versa.

e) Let $\bigcup$ be the universal set and let w$\epsilon \bigcup$.
Let X be an operation such that X(ω) is the value associated with w. let the values of X belong to $\bigcup'$. We say that X is a mapping that maps $\bigcup$ into $\bigcup'$.

i.e X: ⋃ → ⋃ ′
If X maps ⋃ to ⋃ " where ⋃ " ⊂ ⋃ ′ then we say that; ⋃ is the domain of X. ⋃ ′ is the range of X⋃ " is the strict range of X.
Consider tossing a fair coin 3 times and use it to elaborate the above descriptions of Universal Set, Domain, Mapping and Range. (5 Marks)

QUESTION TWO (20 MARKS)

a) Carefully explain the concept of Indicator Functions as used in Probability and Statistics. (5 Marks)
b) Confirm the following properties of Indicator functions
   i. If $A \in B$ then $I_A \leq I_B$ (5 Marks)
   ii. $I_A^C = 1 - I_A$ (5 Marks)
   iii. $I_{(AB)} = I_A I_B$ (5 Marks)

QUESTION THREE (20 MARKS)

a) Prove that Inverse mappings preserves all set relations. (10 Marks)
b) Show that any simple function X can be written as;
$X = \sum_{k=1}^{n} X_K \, I_{AK}$ where X_k's are distinct numerical constants and $A_1, A_2, \dots, A_n$ forms a partition of ⋃, where I_{AK} is an indicator function on sets A and B. (10 Marks)

QUESTION FOUR (20 MARKS)

a) Carefully define Vector Random Variable. (3 Marks)
b) Distinguish the concepts of probability and Probability space as used in Probability and statistics. (5 Marks)
c) Explain the distinguishing item in the classical concept and the Axiomatic concept of probability. (5 Marks)
d) In a bag there are N_1 white, N_2 black and N_3 blue balls. M balls are selected at random without replacement. Find the Laplace order of the experiment. (7 Marks)