MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.

Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411

Fax: 064-30321

Website: www.must.ac.ke Email: info@must.ac.ke

University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR MASTER OF SCIENCE IN STATISTICS

STA 3102: REGRESSION ANALYSIS

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) Given a simple linear regression model $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$

- (i) Obtain the OLS estimator of $\beta_0, \beta_1 \; and \; \beta_2$
- (ii) The estimator of ε_i

(b) Show that the estimator $\hat{\beta}_0, \hat{\beta}_1 \; and \; \hat{\beta}_2$ obtained in a(i) are unbiased estimator of $\beta_0, \beta_1, \beta_2$ respectively .

(c) Obtain $Var(\hat{\beta}_0)$, $Var(\hat{\beta}_1)$, and $Var(\hat{\beta}_2)$,

(d) Test the following hypothesis $H_0 : \beta_0 = 0$
$H_0 : \beta_1 = 0 \; and \; H_0 : \beta_1 = 0$

QUESTION TWO – (20 MARKS)

(a) $y = x\beta + \varepsilon$ show that

(i) $\hat{\beta} = (x'x)^{-1}x'y$

(ii) $\|\varepsilon\|^2 = (y - x\hat{\beta})'(y - x\hat{\beta})$

(b) Given the observation

No.	y	x_1	x_2
1	2	0	2
2	3	2	6
3	2	2	7
4	7	4	5
5	6	4	5
6	8	4	8
7	10	4	7
8	7	6	10
9	8	6	11
10	12	6	9
11	11	8	15
12	14	8	13

Obtain prediction equation of y regressed on x_1 and x_2

QUESTION THREE – (20 MARKS)

(a) Given the model $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots \beta_p x_{pi} + \varepsilon_i$. Write down an ANOVA table for test the hypothesis $H_0 : \beta_1 = \beta_2 = \ldots = \beta_P = 0$

(b) Solve for $\underset{-}{\beta}$

$$\underset{\underset{-}{\beta}}{\arg\min}\; RSS\left(\underset{-}{\beta}\right) = \arg\min\,(y - x\beta)'\Sigma^{-1}(y - x\beta)$$

(c) Solve for $E(\hat{\beta})$ and $Var(\hat{\beta})$ obtained in (b). What is you conclusion.

QUESTION FOUR – (20 MARKS)

Let $y = x\beta + \varepsilon$ with $E(y) = x\beta$ *and* $Var(y) = Var\,(\varepsilon) = \sigma^2 V$, where x is a full-rank matrix and V is a known positive definite matrix. Show that:

(a) The best linear unbiased estimator (BLUE) of β is $\hat{\beta} = \left(x^{/} v^{-1} x\right)^{-1} x^{/} v^{-1} y$

(b) The covariance matrix for $\hat{\beta}$ is $Cor\left(\hat{\beta}\right) = \sigma^2 \left(x^{/} v^{-1} x\right)^{-1}$

(c) An unbiased estimator of σ^2 is $\hat{\sigma}^2 = \dfrac{\left(y - x\hat{\beta}\right)^{/} v^{-1} \left(y - x\hat{\beta}\right)}{n - p - 1}$ where n is the number of observations and p is the number of parameters in the model.

(d) Show that $\hat{\sigma}^2$ in (c) is unbiased estimator of σ^2.

QUESTION FIVE – (20 MARKS)

When gasoline is pumped into a tank of a car, vapour are vented into the atmosphere. An experiment was conducted to determine whether y, the amount of vapour can be predicted using the following four variables based on initial conditions of the tank and the displaced gasoline.

$x_1 = tank\ temperature$
$x_2 = gasoline\ temperature$
$x_3 = vapour\ pressure\ in\ the\ tank$
$x_4 = vapour\ pressure\ of\ gasoline$

Find

(a) $\hat{\beta}$ and $\|\varepsilon\|^2$

(b) $Cor(\hat{\beta})$