MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY

P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3103: STATISTICAL INFERENCE THEORY

DATE: JANUARY 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer Question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Briefly explain the meaning of the following terms/phrases as used in statistical inference
   i. Estimate
   ii. Unbiasedness
   iii. Consistency
   iv. Hypothesis
   v. Type 1 error
   vi. Type 11 error
   vii. Size of a test
   viii. Uniformly Minimum Variance Unbiased Estimator (9 Marks)

b) Let $X_1, X_2, \dots . X_n$ be a random sample from X with density $f(x,\theta), \theta\epsilon\theta_{0,}\theta$ let $0 \leq\propto\leq 1$. consider the testing problem: $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$. Suppose that there exists a test with critical region R of the form.

$$R = \underline{X}: \frac{\prod_{i=1}^{n} f(x_{i,}\theta_1)}{\prod_{i=1}^{n} f(x_{i,}\theta_{0)}} \geq K$$

For some $K \geq 0$ and such that $pr\theta_0(X \in R) = \propto$. Prove that this test is most powerful with size *alpha*. (9 Marks)

c) State without proof, the Rao-Blackwell Theorem and explain its significance in Statistical Inference Theory. (4 Marks)

d) (i) State and prove the Rao-Blackwell-Lehmann-Scheffe Theorem.

(ii) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, \sigma^2)$. Find the UMVUE of θ^2using the Rao-Blackwell-Lehmann-Scheffe Theorem. (8 Marks)

QUESTION TWO (20 MARKS)

a) Assume that all conditions of Crammer-Rao Inequality hold. Show that a necessary and sufficient conditions for an estimator T to be most efficient is that it belongs to a 1-parameter exponential family. (9 Marks)

b) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ). Find an UMVUE of λ^2 and check if there exists and MVBUE for λ^2. (11 Marks)

QUESTION THREE (20 MARKS)

a) Prove the following results

i. If $X_1, X_2, \ldots, X_n$ are iid random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2 \leq \infty$, then a sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is consistent

ii. If T_n *is a sequence of consistent estimators of* θ, and if b_n and C_n are two sequences of numbers such that $\underbrace{lim}_{n\to\infty} b_n = 1$ and $\underbrace{lim}_{n\to\infty} b_n = 0$ then the sequence of estimators is also consistent for θ

(8 Marks)

b) Let $X \approx N(0, \theta)$ and suppose that $T(x) = X^2$. Show that the statistic T(x) is complete. (7 Marks)

c) Suppose $X_1, X_2, \ldots, X_n$ are independently distributed with common mean μ and variance $\sigma_i^2, i + 1,2,3,\ldots, n$. Consider an estimator $\hat{T} = \sum_{i=1}^{n} a_i X_i$. Derive a BLUE $\hat{T}_{\text{BLUE}}$ for μ when $\sigma_i^2 \neq \sigma_2^2 \neq \cdots \neq \sigma_n^2$ and hence conclude that *var*($\hat{T}_{\text{BLUE}}$)= $\frac{1}{\sum_{i=1}^{n}\frac{1}{\sigma_i^2}}$ (5 Marks)

QUESTION FOUR (20 MARKS)

a) Consider a null hypothesis $H_0: \underline{\theta} \epsilon \theta_0$

i. Explain the meaning of Generalised Likelihood Ratio Test (GLRT)

ii. If $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$. Assume that $X \approx N(\mu, \sigma^2)$ unknown. Derive a GLRT for H_0. (15 Marks)

b) i) Briefly distinguish randomized tests from non-randomized tests.

ii) Let $X_1, X_2, \ldots . X_n$ be a random sample from $X \approx N(\mu, \sigma^2)$ with σ^2 known. Derive a Most Powerful test for H_0:$M = M_0$ against $H_1: M_1 \geq M_0$. (5 Marks)