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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3111: STOCHASTIC PROCESS

DATE: DECEMBER 2013 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

a) In a branching process, the probability that any individual has j descendents is given by

$p_n = 0, p_j = \frac{1}{2}, j = 1$

Show that the pgf of the first generation is

$G(s) = \frac{s}{2-s}$

Find the $pgf's$ $G_2(s)$, $G_3(s)$, and $G_4(s)$. Show by induction that

$G_n(s)_2 = \frac{s}{\{2^n-(2^n-1)s\}}$

b) State what is meant by a Markov- chain.

c) A six state Markov – chain with transition max

$$T = 0\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

i. Show that state E_1 is recurrent non-null
ii. Show that state E_3 is transient
iii. Classify the chains in T5.

QUESTION TWO (20 MARKS)

Let $G_n(s)$ pgf of the population size of the n[th] generation of a branching process. The probability that the population size is zero at the nth generation is $G_n(0)$. What is the probability that the population actually becomes extinct at the n[th] generation? Where

$p_j = \frac{1}{2^j+1}, j = 0,1,2,..$

$$G_n(s) = \frac{n}{n+1}\sum_{r=1}^{\infty}\frac{n^{r-1}}{(n+1)^{r+1}}S^r$$

Find the probability of extinction

i. At the nth generation
ii. At the nth generation or later
iii. What is the mean number of generation until extinction occurs?

QUESTION THREE (20 MARKS)

a) Define what is meant by "branching process".
b) Let x_i be the number of children born to an i[th] individual while S_N denote the population size at the N[th] generation. Show that

$var(S_N) = E(N)var(x_i) + var(N), E(x_i^2)$

By

i. Pgf technique
ii. Directly from the definition and the motion of conditional probabilities

QUESTION FOUR (20 MARKS)

a) In a birth and death process with equal birth and death parameters, the probability generating function is

$G(s,t) = \left[\frac{1+(\lambda t-1)(1-s)}{1+\lambda t(1-s)}\right]^{no}$

Find the mean population size at time t. Show also that its variance is $2n_o\lambda t$

b) A three model Markov chain has the transition matrix

$$T = \begin{bmatrix} P & 1-P & 0 \\ 0 & 0 & 1 \\ 1-q & 0 & q \end{bmatrix}$$

Where $0 < p < 1, 0 < q < 1$

Show that the state E_1 is recurrent.

QUESTION FIVE (20 MARKS)

A four state Markov chain has the transition matrix.

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{3}{4} & 0 & \frac{1}{4} & 0 \end{bmatrix}$$

Find f_j the probability that the chain returns at some step to E_j for each state. Determine which states are transient and which states are recurrent. Which state forms a closed subset? Fin the limiting behaviour of $T^n asn \to \infty$