MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY
P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3106: TIME SERIES ANALYSIS

DATE: JANUARY 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer Question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Explain clearly each of the following terms as used in time series analysis
   i. A weekly stationary time series
   ii. Stationarity in the strict sense
   iii. White noise
   iv. Random walk
   v. Autoregressive process (5 Marks)

b) Briefly explain three main objectives of time series analysis. (3 Marks)

c) Define a moving average process. (2 Marks)

d) Find the mean and variance of a MA (q) process. (4 Marks)

e) Consider an AR(2) process given by $X_t = X_{t-1} - \frac{1}{2}X_{t-2} + e_t$. Check whether this process is stationary and if so find its autocorrelation function (ACF). (10Marks)

f) Define spectral density as used in time series and hence derive an expression for the spectral density of a MA(1) process. (6 Marks)

QUESTION TWO (20 MARKS)

a) An observed time series is a mixed ARMA model of the form $X_t = \propto_1 X_{t-1} + \propto_2 X_{t-2} + \cdots + \propto_p X_{t-p} + e_t + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \cdots + \beta_q e_{t-q}$. Using backwards shift operator B express the equation above in terms of $\emptyset_q(B)$ and $\theta_p(B)$

which are polynomials of order q and p respectively.

(6 Marks)

b) Find the Autocovariance function and the Autocorrelation function of AR(1) process converted to an infinite MA process. (14 Marks)

QUESTION THREE (20 MARKS)

a) Suppose that we have data up to time n, $\underline{x}_{1,n} = (x_1, x_2, \ldots x_n)$ and we wish to predict $x_{n+k} (k \geq 1)$. Show that the minimum error forecast of $x_{n+k} (k \geq 1)$ is the conditional mean of x_{n+k} at time n, that is, $\hat{x}(n,k) = E\left(x_{n+k} \middle| \underline{x}_{1,n}\right)$. (16 Marks)

b) Given ARIMA (1,0,0) process as $X_t = \propto X_{t-1} + e_t$, find:

   i. the optimal 2-steps ahead forecast

   ii. the optimal 3-steps ahead forecast (4 Marks)

QUESTION FOUR (20 MARKS)

Consider the following special case of the linear growth model

$X_t = \mu_t + n_t$ where n_t = error, and $\mu_t = \mu_{t-1} + \beta_{t-1}$ with $\beta_t = \beta_{t-1} + \omega_t$.

ω_t and n_t are independent normal with zero means and respective variances σ_ω^2 and σ_n^2.

a) Show that the least squares estimator of the state vector at time $t = 2$ in terms of the observations X_1 and X_2 is $(\hat{\mu}, \quad \hat{\beta}) = (x_2, x_2 - x_1)$ with variance covariance matrix $\rho_2 = \begin{bmatrix} \sigma_n^2 & \sigma_n^2 \\ \sigma_n^2 & 2\sigma_n^2 + \sigma_\omega^2 \end{bmatrix}$ (12 Marks)

b) Derive the Kalman filter estimator of the state vector $at\ t = 3$ when X_3 becomes available and σ_ω^2 is equal to zero. (8 Marks)