



# MERU UNIVERSITY COLLEGE OF SCIENCE & TECHNOLOGY

P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349  
Fax: 020-8027449

## University Examinations 2012/2013

SECOND YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER  
OF SCIENCE IN APPLIED STATISTICS

### STA 3109: MODELLING EXTREMAL EVENTS

DATE: AUGUST 2012

TIME: 3 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions.

#### QUESTION ONE – (30 MARKS)

- Define what is meant by Max – stable distribution. (2 Marks)
- Let  $F$  be a distribution function with right endpoint  $x_f \leq \infty$  and let  $\tau \in (0, \infty)$ . There exists a sequence  $(u_n)$  satisfying  $n\bar{F}(u_n) \rightarrow \tau$  if and only if  $\lim_{x \uparrow x_f} \frac{\bar{F}(x)}{\bar{F}(x-\tau)} = 1$  and  $F(x_f -) = 1$ . Briefly explain the implications of this result. (5 Marks)
- Using the result in b) above, find the limit  $P(M_n \leq u_n) \rightarrow \rho(0,1)$  assuming the;
  - Poisson distribution (5 Marks)
  - Geometric distribution, where  $M_n$  is a maximum point. (4 Marks)
- State some advantages of point process techniques in the analysis of extremes. (3 Marks)
- Describe the Gumbel method of exceedances as used to explore data for extremes. (5 Marks)
- Outline the process of fitting the Generalised Pareto Distribution to excesses over a threshold. (6 Marks)

#### QUESTION TWO (20 MARKS)

- What is meant by tail-equivalent? (2 Marks)
- State and prove the Fishere – Tippett theorem as a limit law for maxima. (7 Marks)

- c) Describe the upper tail characteristics of the three resultant distributions under the theorem in b) above and give examples of the standard distributions which can apply each of these limiting distribution laws; (6 Marks)
- i. Gumbel
  - ii. Fréchet
  - iii. Weibull
- d) Using the Fisher-Tippet theorem derive an expression for computing the Maxima of exponential random variables. (5 Marks)
- e) Consider the application of extreme value modeling in hydrology, what are some of the questions that can be solved by this methodology? (3 Marks)

**QUESTION THREE (20 MARKS)**

- a) Clearly define what is meant by a mean excess function of a random variable, say  $X$ . (2 Marks)
- b) Describe how the mean excess function can be calculated mathematically, using the partial integration. (3 Marks)
- c) Define the Generalized Pareto Distribution (GPD). (2 Marks)
- d) Clearly outline the properties of a GPD. (13 Marks)

**QUESTION FOUR (20 MARKS)**

- a) Discuss how to explore data for extreme using probability and quantile plots. (4 Marks)
- b) Describe the properties of QQ-plots which are in essence the merits of these plots while exploring data for extremes. (8 Marks)
- c) Discuss the return period method of analyzing extremal events. Provide relevant example(s). (8 Marks)