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University Examinations 2012/2013

FIRST YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3132: ANALYTICAL APPLIED MATHEMATICS 1

DATE: AUGUST 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

- **The variables used have their usual meaning**

QUESTION ONE – (30 MARKS)

a) State any three Dirichlet conditions necessary for a function to be expressed as a Fourier series. (3 Marks)

b) Evaluate the integral $\oint_c (x+2y)dx + (y-2x)dy$ around the ellipse c defined by $x = 4\cos\theta, y = 3\sin\theta \; for \; 0 \leq \theta < 2\pi$ and c is described in a clockwise direction. (4 Marks)

c) By transforming from Cartesian $x_1 = \{x, y, z\}$ to cylindrical $\overline{x_i} = \{r, \theta, l\}$ coordinates, obtain the components of the metric tensor g_{ij} and its inverse g^{ij} in cylindrical coordinates. (5 Marks)

d) Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{2-\cos\theta}$. (6 Marks)

e) Use the Laplace transform method to solve the I.V.P
$y'' - 10y' + 9y = 5t, \;\; y(0) = -1, y'(0) = 2.$ (6 Marks)

f) Verify that the functions $f_1(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$ and $f_2(z) = \sum_{n=1}^{\infty} \frac{(z-2i)^{n-1}}{(2-i)^n}$ are analytic continuations of each other hence sketch the common region. (6 Marks)

QUESTION TWO (20 MARKS)

a) Identify the zeros and singularities of the function $f_2 = \frac{2z^2+1}{z^2+1}$ (2 Marks)

b) Using the tensor identity $\varepsilon_{ijk}\,\varepsilon_{ipq} = \delta_{jp}\,\delta_{kq} - \delta_{jq}\,\delta_{kp}$ prove the vector identity
$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$ (8 Marks)

c) Use the residue theorem (or otherwise) to evaluate the integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\,d\theta$
(10 Marks)

QUESTION THREE (20 MARKS)

a) Find a Laurent series expansion of $f(z) = \frac{1}{(z+1)(z-3i)}$ about $z = -1$ in the punctured disc
$0 < |z+1| < \sqrt{10}$. (9 Marks)

b) Evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2}\,dx$ (11 Marks)

QUESTION FOUR (20 MARKS)

a) Calculate the residues of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at each of its singularities
hence (or otherwise) evaluate the integral $f(z) = \oint_c \frac{z^2-2z}{(z+1)^2(z^2+4)}\,dz$ where $c\colon |z| = 3$.
(9 Marks)

b) Use the Laplace transform method to solve the equation $2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = te^{-2t}$
subject to $y(0) = 0 \quad y'(0) = -2$. (11 Marks)