## University Examinations 2011／2012

FIRST YEAR，FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3132：ANALYTICAL APPLIED MATHEMATICS
DATE：AUGUST 2011
TIME： 3 HOURS
INSTRUCTIONS：Answer question one and any other two questions．
QUESTION ONE－（30 MARKS）
（a）Determine the residue of $f(Z)=\frac{1}{1+\mathbb{Z}^{4}}$ at each of its poles in a finite $Z-$ plane．
（8 Marks）
（b）Find in finite $\oint f(Z) d Z$ where $c$ is the unit circle $|Z|=1$ and

$$
\begin{equation*}
f(Z)=\frac{Z^{2}+1}{(Z-2)(2 Z+1)^{2}(2 Z-1)} \tag{6Marks}
\end{equation*}
$$

（c）Find $g$ and $g^{i i}$ corresponding to the metric
（i）$d s^{2}=5\left(d x^{1}\right)^{2}+3\left(d x^{2}\right)^{2}+4\left(d x^{3}\right)^{2}-6 d x^{1} d x^{2}+4 d x^{2} d x^{3}$
（ii）$d s^{2}=\frac{d r^{2}}{1-r^{2} / R^{2}}+r\left(d \theta^{2}+\sin ^{2} \theta d \emptyset^{2}\right)$ where R is a constant．
（d）Use the Laplace transform method to solve
$y^{\prime \prime}-3 y^{\prime}+2 y=e^{-t}$ ，given $y(0)=1, y^{\prime}(0)=0$

## QUESTION TWO－（ 20 MARKS）

（a）Evaluate the contour integral

$$
\begin{equation*}
\oint \frac{d Z}{Z^{3}\left(Z^{2}+2 Z+2\right)}, \text { where } \mathrm{c} \text { is the circle }|Z|=3 . \tag{12Marks}
\end{equation*}
$$

(b) Evaluate $\int_{-\infty}^{\infty} \frac{d x}{\left(1+x^{2}\right)^{3}}$

## QUESTION THREE - (20 MARKS)

(a) Evaluate using the Residue theorem, $\int \frac{d \theta}{3+2 \cos \theta}$
(b) A contravariant tensor has components $a, b, c$ in rectangular coordinate system. Find the components in spherical coordinate system.

## QUESTION FOUR - (20 MARKS)

Use the complex form of the fourier transform to show that
$u(x, t)=\frac{1}{2 \sqrt{\pi k t}} \int_{-\infty}^{\infty} f(\xi) e^{-(x-\xi)^{2} / 4 k t} d \xi$ is a solution of the boundary value problem below, governing the heat conduction in a very long metal bar which extends from $-\infty$ to $\infty$
$u_{t=K U_{x x}}, \quad u(x, 0)=f(x),-\infty<x<\infty$

