

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3132: ANALYTICAL APPLIED MATHEMATICS

DATE: AUGUST 2011

TIME: 3 HOURS

(8 Marks)

INSTRUCTIONS: Answer question one and any other two questions.

QUESTION ONE – (30 MARKS)

(a) Determine the residue of $f(Z) = \frac{1}{1 + \mathbb{Z}^4}$ at each of its poles in a finite Z – plane.

- (b) Find in finite $\oint f(Z)dZ$ where *c* is the unit circle |Z| = 1 and $f(Z) = \frac{Z^2 + 1}{(Z-2)(2Z+1)^2(2Z-1)}$ (6 Marks)
- (c) Find g and g^{ii} corresponding to the metric

(i) $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$

(ii)
$$ds^2 = \frac{dr^2}{1 - r^2/R^2} + r(d\theta^2 + \sin^2\theta d\phi^2)$$
 where R is a constant. (11 Marks)

(d) Use the Laplace transform method to solve

$$y'' - 3y' + 2y = e^{-t}$$
, given
 $y(0) = 1, y'(0) = 0$ (5 Marks)

QUESTION TWO – (20 MARKS)

(a) Evaluate the contour integral $\oint \frac{dZ}{Z^3(Z^2+2Z+2)}, \text{ where c is the circle } |Z| = 3.$ (12 Marks)

(b) Evaluate
$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$$
 (8 Marks)

QUESTION THREE – (20 MARKS)

(a) Evaluate using the Residue theorem,

$$\int \frac{d\theta}{3+2\cos\theta}$$
(5 Marks)

(b) A contravariant tensor has components *a*, *b*, *c* in rectangular coordinate system. Find the components in spherical coordinate system. (15 Marks)

QUESTION FOUR – (20 MARKS)

Use the complex form of the fourier transform to show that

 $u(x,t) = \frac{1}{2\sqrt{\pi kt}} \int_{-\infty}^{\infty} f(\xi) e^{-(x-\xi)^{2/4kt}} d\xi$ is a solution of the boundary value problem below, governing the heat conduction in a very long metal bar which extends from $-\infty$ to ∞

 $u_{t=KU_{xx}}, \quad u(x,0)=f(x), -\infty < x < \infty$

(20 Marks)