MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.

Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411

Fax: 064-30321

Website: www.must.ac.ke Email: info@mucst.ac.ke

University Examinations 2012/2013

FIRST YEAR, FIRST/SECOND SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3132: ANALYTICAL APPLIED MATHEMATICS I

DATE: APRIL 2013 **TIME: 3HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

- **The variables and symbols have their usual meaning.**

QUESTION ONE - (30 MARKS)

a) Identify the zeros and singularities of the following complex functions hence give the order of the zeros and classify the singularities.

   i. $f(z) = \frac{(z-i)^2}{z+1}$ (3 Marks)

   ii. $f(z) = \sin\left(\frac{1}{z^2}\right)$. (3 Marks)

b) Evaluate the integral $\int_0^{\pi} \frac{1}{5+3\cos\theta} d\theta$. (6 Marks)

c) Identify the singularities of the function $f(z) = \frac{\sin z}{(z-i)(z+1)^2}$ hence calculate the residues at the poles. (5 Marks)

d) Use tensor algebra to prove that $\vec{a}.\left(\vec{b} \times \vec{c}\right) = \vec{b}.\left(\vec{c} \times \vec{a}\right) = \vec{c}.\left(\vec{a} \times \vec{b}\right)$. (4 Marks)

e) Solve the I. V. P $y'' - 10y' + 9y = 5x$ subject to $y(0) = -1, y'(0) = 2$ using the Laplace transform method. (7 Marks)

QUESTION TWO (20 MARKS)

a) By transforming from Cartesian $x_i = \{x, y, z\}$ to cylindrical $\bar{x}_i = \{r, \theta, l\}$ coordinates, obtain the components of the metric tensor g_{ij} and the inverse g^{ij} in cylindrical coordinates. (5 Marks)

b) Show that for some real number $a > 1$ then the integral $\int_0^{\pi} \frac{d\theta}{a+\cos\,\theta} = \frac{\pi}{\sqrt{a^2-1}}$.
(7 Marks)

c) State and prove the Cauchy's theorem. (8 Marks)

QUESTION THREE (20 MARKS)

a) Find the Laurent series expansion of $f(z) = \frac{1}{z^2+(1-3i)z-3i}\ about\ z = 3i$ and state the disc of convergence. (6 Marks)

b) Evaluate the integrals

i. $\int_{-\infty}^{\infty} \frac{x^2+2}{x^4+13x^2+36} dx$ (8 Marks)

ii. $\oint_c \frac{2z}{(z-2)(z+1)(z-i)} dz\ \ c: |z| = 3$ (6 Marks)

QUESTION FOUR (20 MARKS)

a) Let $\emptyset$ be a scalar field while $\vec{u}$ and $\vec{v}$ are vector fields. Prove the following expression.

i. $\vec{\nabla}.(\emptyset\,\vec{u}) = \vec{u}.\vec{\nabla}\emptyset + \emptyset\vec{\nabla}.\vec{u}$ (5 Marks)

ii. $\vec{\nabla} \times (\vec{u} \times \vec{v}) = (\vec{\nabla}.\vec{v})\vec{u} - (\vec{\nabla}.\vec{u})\vec{v} + (\vec{v}.\vec{\nabla})\vec{u} - (\vec{u}.\vec{\nabla})\vec{v}$ (7 Marks)

b) Use the Laplace transform method to solve the equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = 1 + e^{-t}$ subject to $y(0) = 0, y'(0) = 0$. (8 Marks)