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University Examinations 2013/2014

FIRSTYEAR, SECOND SEMESTER EXAMINATIONS FOR DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3132: ANALYTICAL APPLIED MATHEMATICS I

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) By giving an appropriate example, state (illustrate) what is a Dirichlet problem. (3 Marks)

(b) Determine the Fourier Sine and Fourier Cosine transforms of the first derivative of a function $f(x)$. Explain why they cannot be used to transform a P.D.E into an O.D.E. (8 Marks)

(c) Let $F(w)$ and $G(w)$ be the Fourier transforms of $f(x)$ and $g(x)$, obtain the convolution of $g(x)$ and $f(x)$ (6 Marks)

(d) Show that $\oint_c \overline{f(z)}\, dz = (circulation) + i(net\ flux)$ of a fluid flow, where c is a positively oriented, simple closed contour c. (7 Marks)

(e) Prove that $\dfrac{\partial g_{pq}}{\partial x^m} = -g^{pn}\begin{Bmatrix} q \\ mn \end{Bmatrix} - g^{qn}\begin{Bmatrix} p \\ mn \end{Bmatrix}$ (6 Marks)

QUESTION TWO – (20 MARKS)

(a) Given that $\mathcal{L}\{\delta(t-b)\} = e^{-sb}$ for $b > 0$, and $F(s)$ and $G(s)$ are Laplace transforms of functions $f(t)$ and $g(t)$ respectively. Determine the convolution theorem of $f(t)$ and $g(t)$. (6 Marks)

(b) Given a simple closed curve c oriented counter clockwise in the plane and a complex function $f(z)$. Discuss the meaning of net flux across C being

(i) Positive

(ii) Negative

(iii) Zero (4 Marks)

(c) Use the appropriate Fourier integral transform to solve the given boundary –value problem. (10 Marks)

$$u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, \ y > 0$$

$$u(0, y) = f(y), \ \left.\frac{\partial u}{\partial x}\right|_{x=\pi} = 0, \ y > 0$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \quad 0 < x < \pi$$

QUESTION THREE – (20 MARKS)

(a) Explain a multiply connected domain. Hence with aid of a diagram illustrate how to make a doubly connected domain become a simply connected one. (8 Marks)

(b) In a coordinate system x^i, $A(P, q, r)B_r^{qS}$ where B_r^{qS} is an arbitrary tensor and $C_p^{\ s}$ is a tensor. Show that $A(P, q, r)$ is also a tensor. (12 Marks)

QUESTION FOUR – (20 MARKS)

(a) Let $w = f(z)$ be an analytic mapping of a domain D in the z-plane onto a domain D' in the w-plane. Outline the method of solving Dirichlet problems, using the domain D'. (7 Marks)

(b) Use displacement of a semi-infinite elastic beam is given by

$$a^2 u_{xx} = u_{tt}, \; x > 0, \; t > 0$$

$u(0,t) = f(t), \; \lim_{x \to \infty} \frac{\partial u}{\partial x} = 0, \; t > 0$. Solve for $u(x,t)$ by Laplace transform.

$$u(x,0), \; \left.\frac{\partial y}{\partial t}\right|_{t=0}, \; t > 0$$

(13 Marks)

QUESTION FIVE – (20 MARKS)

(a) Find the infinite Hankel transform of $\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx} - \frac{n^2}{x^2} f$ (16 Marks)

(b) Determine the function whose double transform is $e^{-kw^2 t}$ (4 Marks)