MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY
P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2012/2013

FIRST YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3106: COMPLEX ANALYSIS

DATE: AUGUST 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

- **The variables used have their usual meaning**

QUESTION ONE – (30 MARKS)

a) Prove that a necessary condition for a complex function to be harmonic in a domain D is that it has to be analytic there. (8 Marks)

b) Distinguish between a conformal and an isogonal mapping. (2 Marks)

c) Formulate a bilinear transformation which maps the points i, -i and 1 of the z plane into 0, 1 and ∞ of the w plane respectively. (4 Marks)

d) Test if the function $4z^2 - 3z + 2$ satisfies the Schwarz reflection principle. (3 Marks)

e) Prove that the function $v(x, y) = e^{-x}(x \cos y + y \sin y)$ is harmonic hence find its conjugate function. (7 Marks)

f) Show that if f(z) is analytic in the upper half plane of the z-plane and $z_0 = x_0 + iy_0$ is any point inside the upper half plane then $f(z_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{y_0 f(x)}{(x-x_0)^2+y_0^2} dx$ (6Marks)

QUESTION TWO (20 MARKS)

a) Test analytically of $f(z) = \frac{3z^2-2}{z}$ stating the region of analyticity. (5 Marks)

b) Explain the concept of a Schwarz – christofel transformation. (2 Marks)

c) Compute the product $\prod_{n=1}^{\infty} \left[\frac{n+1+(-1)^n}{n+1}\right]$ (5 Marks)

d) Prove that a necessary and sufficient condition for the product $\prod_{n=1}^{\infty}(1+|u_n|)$ to converge is that the series $\sum_{n=1}^{\infty}(u_n) < \infty$. (8 Marks)

QUESTION THREE (20 MARKS)

a) Find the image of lines parallel to the imaginary axis of the z-plane under the transformation$f(z) = z^2$. (8 Marks)

b) Find and plot the image of the region bounded by the straight lines $L_1: x = 1, L_2: y = 2$ and $L_3: y = x + 2$ under the mapping $w = z^2 - 4z + 4$ hence test if this mapping is conformal at the vertex where the line L_1 meetsL_2. (12 Marks)

QUESTION FOUR (20 MARKS)

a) Construct a linear fractional transformation mapping the points $z_1 = 1, z_2 = i, z_3 = 0$ to $w_1 = 5 + i, w_2 = 1 - i, w_3 = -i$ (7 Marks)

b) Given that f(z) is analytic inside and on a circle $c: |z| = R$ and $z = re^{i\theta}$ is any point inside c, then

$$u(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi}\frac{R^2 - r^2}{R^2 + r^2 - 2Rr\cos(\theta - \emptyset)}u(R,\emptyset)d\emptyset$$ and

$$v(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi}\frac{R^2 - r^2}{R^2 + r^2 - 2Rr\cos(\theta - \emptyset)}v(R,\emptyset)d\emptyset$$

Where $u(r,\theta)$ and $v(r,\theta)$ are the real and imaginary parts of f(z) while $u(R,\emptyset)$and $v(R,\emptyset)$ are the real and imaginary parts of $f(Re^{i\emptyset})$ (13 Marks)