### University Examinations 2013/2014

FIRSTYEAR, SECOND SEMESTER EXAMINATIONS FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

### SMA 3106: COMPLEX ANALYSIS I

**DATE: APRIL 2014**                                    **TIME: 3 HOURS**

**INSTRUCTIONS:** *Answer question **one** and any other **two** questions.*

**QUESTION ONE – (30 MARKS)**

(a) Distinguish between an ordinary point and a singular point.          (2 Marks)

(b) Define a conformal mapping.                                    (2 Marks)

(c) State the maximum modulus principle.                            (2 Marks)

(d) Identify and classify the singularities of the function $f(z) = \dfrac{z^2}{\left(1 + z^2\right)^2}$ hence or otherwise

calculate the residue of $f(z)$ at one of the poles.                    (5 Marks)

(e) Test if the function $u = \dfrac{x}{|z|^2}$ is harmonic hence find its conjugate function such that

$f(z) = u + iv$ is analytic.                                    (6 Marks)

(f) Construct a bilinear transformation that maps the points $z_1 = 1,\ z_2 = -i\ and\ z = 0$ from
the z-plane to the points $w_1 = 5 + i,\ w_2 = 1 - i\ and\ w_3 = -i$ in the w-plane.  (6 Marks)

(g) Find the image of horizontal straight lines $\text{Im}(z) = a$ under an inversion. $(a \neq 0)$

(5 Marks)

(h) Test the analyticity of $f(z) = \tan^{-1}\left(\dfrac{y-b}{x-a}\right)$ (2 Marks)

## QUESTION TWO – (20 MARKS)

(a) Expand the function $f(z) = \dfrac{1}{z^2 - 3iz + z - 3i}$ about $z = -1$ using Laurent series hence state the disc of convergence. (6 Marks)

(b) Evaluate each of the following integrals

(i) $\displaystyle\oint_c \dfrac{\ln z^3}{z+2} dz$ for $c : |z| = 3$ (6 Marks)

(ii) $\displaystyle\int_{-\infty}^{\infty} \dfrac{x \sin x}{\left(x^2 + a^2\right)^2}$ (8 Marks)

## QUESTION THREE – (20 MARKS)

(a) Express $z^\alpha$ as a logarithmic function hence evaluate $i^{-i}$ where $\alpha \neq 0$ (4 Marks)

(b) Test if the function $f(z) = 3z^2 + 4 - 2i$ satisfies the Shwarz reflection principle.

(3 Marks)

(c) (i) State the Cauchy's residue theorem. (3 Marks)

(ii) use the residue theorem to evaluate the integral $\displaystyle\int_0^{2\pi} \dfrac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ (10 Marks)

## QUESTION FOUR – (20 MARKS)

(a) Define each of the following
   (i)  Entire function (2 Marks)
   (ii) Compact set (2 Marks)

(b) Calculate the residues of the function $f(z) = \dfrac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at its poles. (5 Marks)

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(c) State and prove the Poissons integral formula for a circle given as

$$f\left(re^{i\theta}\right)=\frac{1}{2\pi}\int_0^{2\pi}\frac{R^2-r^2}{R^2-r^2-2rR\cos(\theta-\Phi)}f\left(\mathrm{Re}^{i\Phi}\right)d\Phi \quad \text{with the variables used having}$$

their usual meaning. (11 Marks)