MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY
P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2011/2012

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3106: COMPLEX ANALYSIS I

DATE: AUGUST 2011 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) Express z^k as a logarithmic function hence simplify the expression $(-i)^{3i}$ (4 Marks)

(b) Prove that the function $u(x,y) = e^{-x}(x\ \sin y - y\cos y)$ is harmonic hence find the conjugate function of u. (8 Marks)

(c) Prove that for closed polygons whose interior angles are $a_1, a_2, \ldots a_n$ we have the sum
$$\left(\frac{a_1}{\pi} - 1\right) + \left(\frac{a_2}{\pi} - 1\right) + \cdots + \left(\frac{a_n}{\pi} - 1\right) = -2$$ (3 Marks)

(d) Calculate the residues of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at each of its singularities hence (or otherwise) evaluate the integral
$f(z) = \oint \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ where $c\!: |z| = 3$ (8 Marks)

(e) Show that a function $f(z) = z^2$ maps lines parallel to the real axis in the z plane to parabolas in the w plane. Sketch these parabolas. (7 Marks)

QUESTION TWO – (20 MARKS)

(a) Show that the functions $f_1(z) = \sum_{n=0}^{\infty} \frac{(z+2i)^n}{(1+2i)^{n+1}}$ and $f_2(z) = \sum_{n=0}^{\infty} z^n$ are analytic continuations of each other hence sketch the common region. (8 Marks)

(b) State and prove the Poisson's integral formula for a circle given as $f\left(re^{i\theta}\right) = \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2-r^2}{R^2+r^2-2Rr\cos(\theta-\emptyset)} f\left(Re^{i\emptyset}\right)d\emptyset$ where the given variables have their usual meaning. (12 Marks)

QUESTION THREE – (20 MARKS)

(a) Test if the function $f(z) = 3z^2 + 4 - 2i$ satisfies the Schwarz reflection principle. (3 Marks)

(b) Show that under an inversion points inside a circle in the Z-Plane are mapped to points outside a circle in the W-Plane. (9 Marks)

(c) Find a Laurent series expansion of $f(z) = \frac{1}{(z+1)(z-3i)}$ about $z = -1$ in the punctured disc $0 < |z+1| < \sqrt{10}$ (8 Marks)

QUESTION FOUR – (20 MARKS)

(a) Define the following terms with respect to complex functions

(i) Isogonal (2 Marks)

(ii) Meromorphic (2 Marks)

(b) Prove that a necessary condition for a function to be harmonic in a region is that it is analytic in that region. (9 Marks)

(c) Construct a linear fractional transformation mapping the points $z_1 = 1 \;\; z_2 = -i \; and \;\; z_3 = 0 \; to \; w_1 = 5 + i \;\; w_2 = 1 - i \; and \; w_3 = -i$ (7 Marks)