



# MERU UNIVERSITY COLLEGE OF SCIENCE & TECHNOLOGY

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## University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN  
APPLIED MATHEMATICS

### SMA 3134: FLUID MECHANICS I

DATE: DECEMBER 2012

TIME: 3 HOURS

INSTRUCTIONS: Answer questions *one* and any other *two* questions

- The variables have their usual meaning.

#### QUESTION ONE - (30 MARKS)

- a) Distinguish between a streamline and a pathline as used in fluid mechanics. (2 Marks)
- b) Define a compressible flow and show that for such flows  $\vec{\nabla} \cdot \vec{q} = 0$  (4 Marks)
- c) Define a cycle as used in relation to heat engines. (2 Marks)
- d) A fluid (gas) is defined such that for a unit mass  $P = \frac{u}{4V}$ . Verify that  $c_v = 4V \left( \frac{\partial P}{\partial T} \right)_v$  and  $c_p = 5p \left( \frac{\partial v}{\partial T} \right)_p$  (5 Marks)
- e) Show that the mass conservation equation of a two dimensional irrotational fluid flow of a compressible fluid can be written as.  
$$\frac{\partial^2 \phi}{\partial t^2} + \vec{q} \cdot \frac{\partial \vec{q}}{\partial t} + \frac{a^2}{\rho} \frac{\partial \rho}{\partial t} = 0$$
 where  $\phi$ ,  $\vec{q}$  and  $a$  are scalar functions, fluid velocity and speed of sound respectively. (7 Marks)
- f) Briefly discuss the flow of a compressible fluid through a converging diverging nozzle. (6 Marks)
- g) Define a mach cone and how it generated in the flow of a fluid. (4 Marks)

#### QUESTION TWO – (20 MARKS)

- a) State the Zeroth law of thermodynamics. (2 Marks)

- b) Test if the quantity of heat  $Q = Q(P, V)$  added to a unit mass of a perfect gas is a function of state. (8 Marks)
- c) State and prove the Carnot's theorem. (10 Marks)

**QUESTION THREE - (20 MARKS)**

A perfect gas is initially at a state A at pressure  $P_1$  and temperature  $T_1$ . It is then cooled at constant pressure to a state B. The gas is expanded adiabatically to a state C at a pressure  $P_2$  and volume  $V_2$ . It is then heated at constant volume to a state D before finally being compressed adiabatically back to state A.

- a) Draw an indicator diagram for the cycle. (2 Marks)
- b) Show that the heat per unit mass absorbed from the hot source along CD is
- $$Q = \frac{\gamma R}{\gamma - 1} \left\{ T_2 \left( \frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} - T_1 \right\} \quad (8 \text{ Marks})$$
- c) Compute the efficiency of the cycle. (10 Marks)

**QUESTION FIVE – (20 MARKS)**

- a) An ideal gas moving at a free stream supersonic speed  $U_1$  experiences a normal shock and moves at a speed  $U_2$ . Show that for this gas

$$\frac{\rho_2}{\rho_1} = \frac{U_1}{U_2} = \frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2} \quad (7 \text{ Marks})$$

- b) The flow past a stationary cylinder of radius  $R$  can be modeled as a superposition of a uniform flow of velocity  $U$  and a doublet of strength  $\mu$ . Show the following

i. The maximum velocity of the flow is at the surface  $r = R$  where  $\theta = \pm \pi/2$  (6 Marks)

ii. The pressure  $P = P^* + \frac{\rho u^2}{2} (1 - 4 \sin^2 \theta)$  where  $P^*$  is the free stream pressure. (3 Marks)

iii. The lift  $L = 0$  (4 Marks)