

# **University Examinations 2011/2012**

# FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

### SMA 3134: FLUID MECHANICS I

DATE: AUGUST 2011

**TIME: 3 HOURS** 

**INSTRUCTIONS:** Answer question one and any other two questions.

## **QUESTION ONE – (30 MARKS)**

(a) Define the following terms as used in fluid mechanics.	
(i) Magnetohydrodnamics	(2 Marks)
(ii) Mach cone	(2 Marks)
(iii)Potential flow	(2 Marks)
(b) State the first law of thermodynamics	(2 Marks)
(c) Differentiate between a lift and a drag.	(2 Marks)

- (d) Show that the quantify of heat Q = Q(P, V) added to a unit mass of a perfect gas is not a function of state. (7 Marks)
- (e) By modeling the flow past a stationary cylinder of radius R as a superposition of a uniform flow of velocity U, a doublet of strength  $\mu$  show that
  - (i) The maximum velocity is a the surface r = R when  $\theta = \pm \frac{\pi}{2}$  (6 Marks)

(ii) 
$$P = P^* + \frac{\rho U^2}{2} (1 - 4sin^2\theta)$$
 where  $P^*$  is the free stream pressure (3 Marks)

(iii) The lift 
$$L = 0$$
 (4 Marks)

#### **QUESTION TWO – (20 MARKS)**

- (a) Show that for a perfect gas  $dS = \frac{c_p}{V} dV + \frac{c_v}{P} dP$  and the entropy per unit mass of the gas is a function of state and that for isentropic fluid flows,  $PV^{\gamma} = Cons \tan t$  (10 Marks)
- (b) State and prove that Carnot's theorem. (10 Marks)

#### **QUESTION THREE – (20 MARKS)**

A perfect gas is initially in a state A at pressure  $P_1$  and temperature  $T_1$ . It is expanded adiabatically to a state B. It is then cooled at constant volume to a state C at a pressure  $P_2$  and temperature  $T_2$ . It is then compressed adiabatically to a state D at pressure  $P_1$ . Finally it is heated at constant pressure back to state A.

(a) Show that the heat per unit mass absorbed from the hot source along DA is

$$Q = \frac{\gamma R}{\gamma - 1} \left\{ T_1 - T_2 \left( \frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} \right\}$$
(9 Marks)

Show that the efficiency of the cycle is  $\eta = 1 + \frac{T_2 - T_1 \left(\frac{P_2 T_1}{P_1 T_2}\right)^{r}}{\gamma \left\{T_1 - T_2 \left(\frac{P_1}{P_2}\right)^{\frac{r}{\gamma}}\right\}}$ 

(b)

(11 Marks)

### **QUESTION FOUR - (20 MARKS)**

Show that for a two dimensional irrotational fluid flow of a compressible fluid we have the vorticity along the z axis as

$$\omega = \frac{1}{\rho} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{1}{\rho a^2} \left\{ u^2 \frac{\partial^2 \psi}{\partial x^2} + 2uv \frac{\partial^2 \psi}{\partial x \partial y} + v^2 \frac{\partial^2 \psi}{\partial y^2} \right\} + \frac{1}{\rho c_p} \frac{dS}{d\psi} \left[ \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right]$$
(20 Marks)