

University Examinations 2012/2013

FIRST YEAR, SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3134: FLUID MECHANICS 1

DATE: AUGUST 2012

TIME: 3 HOURS

INSTRUCTIONS: Answer question one and any other two questions.

• The variables used have their usual meaning

QUESTION ONE – (30 MARKS)

- a) Define the "velocity of sound" as used in fluid mechanics. (2 Marks)
- b) State the Carnot's theorem. (2 Marks)
- c) State the equations governing the steady two dimensional flow of an inviscid thermally non conducting fluid. (3 Marks)
- d) Given that the internal energy u, the pressure P, and the volume V of a unit mass is given by u=4PV,
 - i. Show that $C_V = 4V \left(\frac{\partial P}{\partial T}\right)_V$ and $C_P = 5P \left(\frac{\partial V}{\partial T}\right)_P$. (4 Marks)
 - ii. Given that $\gamma = \frac{C_P}{C_V}$ is a constant show that $P^{4\gamma}V^5 = F(T)$ where F(T) is a function of temperature only. (7 Marks)
- e) Define the Mach number (M) hence classify the various types of fluid flow in terms of the mach number. (6 Marks)
- f) What is an isothermal, adiabatic and isentropic process? (4 Marks)
- g) State the second law of thermodynamics. (2 Marks)

QUESTION TWO (20 MARKS)

- a) Distinguish between compressible and incompressible fluids. (2 Marks)
- b) The pressure, volume, temperature, internal energy and entropy of a given mass in equilibrium are P,V,T,U and S. Prove that

$$\frac{\partial U}{\partial T} = T \frac{\partial S}{\partial T} and$$

$$\frac{\partial U}{\partial V} = T \frac{\partial S}{\partial V} - P = T \frac{\partial P}{\partial T} - P$$
(8 Marks)

c) Show that the velocity distribution of a fluid passing through a converging – diverging nozzle of cross section A is given by $u^2 = 2a^2 \ln \left[\frac{Au}{A_0u_0}\right] + u_0^2$ where u_0 and A_0 are the neck velocity and cross sections respectively. (10 Marks)

QUESTION THREE (20 MARKS)

- a) Show that the energy equation for a one dimensional compressible fluid flow is given by $\frac{\gamma}{\gamma-1}\frac{P}{\rho} + \frac{1}{2}u^2 = \frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0} + \frac{1}{2}u_0^2 \text{ for some reference values of } P_0, \rho_0 \text{ and } u_0. \quad (8 \text{ Marks})$
- b) Prove that the pressure corresponding to the critical velocity of sound is

$$P^* = P_0 \left(\frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$
(12 Marks)

QUESTION FOUR (20 MARKS)

A perfect gas is initially in a state A at pressure P_1 and temperature T_1 . It is expanded adiabatically to a state B. It is then cooled at constant volume to a state C at a pressure P_2 and temperature T_2 . It is then compressed adiabatically to a state D at pressure P_1 . Finally it is heated at constant pressure back to state A.

a) Show that the heat per unit mass dissipated to the cold sink along BC is

$$Q = \frac{R}{\gamma - 1} \left\{ T_2 - T_1 \left[\frac{P_2 T_1}{P_1 T_2} \right]^{\gamma - 1} \right\}$$
(9 Marks)

b) Show that the efficiency of the cycle is $\eta = 1 - \frac{\gamma \left\{ T_1 - T_2 \left[\frac{P_1}{P_2} \right]^{\frac{\gamma}{\gamma}} \right\}}{T_2 - T_1 \left[\frac{P_2 T_1}{P_1 T_2} \right]^{\gamma-1}}$ (11 Marks)