



**MASENO UNIVERSITY**  
**UNIVERSITY EXAMINATIONS 2015/2016**

**FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR  
THE DEGREE OF MASTER OF SCIENCE IN APPLIED  
STATISTICS**

**CITY CAMPUS**

**SAS 808: SAMPLING METHODS**

Date: 22<sup>nd</sup> May, 2016

Time: 9.00 - 12.00 noon

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**INSTRUCTIONS:**

- Answer question ONE and any other TWO questions.

**Question One (24 marks)**

- (a) In a population with  $N=6$  the values of  $x_i$  are 5,7,4,12 and 8. Calculate  $s^2$  for all simple random samples of size 2 and verify that  $E(s^2) = S^2$   
(6 marks)
- (b) A simple random sample of size 3 is drawn from a population of size  $N$  with replacement. Find the probabilities that the sample contains 1,2,3 different units ( that is aaa,aab,abc)  
( 6 marks)
- (c) Give and explain three factors you should consider picking when deciding the size of the sample which should be picked from a stratum  
( 6 marks)
- (d) Give and prove the conditions in which the mean of a systematic sampling is more precise than the mean of Simple random sampling  
(6 marks)

**Question 2 (18 marks)**

- (a) In a population with  $N=6$  and strata =2 values  $x_{hi}$  are 0,1,2 in stratum 1 and 4,6,11 in stratum 2. A sample of size 4 is to be taken. Show that the optimum  $n_{hi}$  when rounded to integers are  $n_{h1} = 1$  in stratum 1 and  $n_{h2} = 3$  in stratum 2  
(10 marks)
- (b) Give and Prove conditions when
- (i) Simple random sampling will be better than Stratified sampling
  - (ii) Simple random sampling will be better than Systematic sampling.
- (10 marks)

**Question 3 (18 marks)**

- (a) If the cost function is of the form  $C = c_0 + \sum t_h \sqrt{n_h}$  where  $c_0$  and  $t_h$  are known numbers, show that in order to minimize  $V(\bar{x}_{st})$  for fixed total cost  $n_h$  must be proportional to  $\left(\frac{W_h^2 S_h^2}{t_h}\right)^{\frac{2}{3}}$
- (b) If  $V_{prop}(\bar{x}_{st})$  is the variance of the estimated mean from a stratified random sample of size  $n$  with the proportional allocation and  $V(\bar{x})$  is the variance of the mean of a simple random sample show that the ratio  $\frac{V_{prop}(\bar{x}_{st})}{V(\bar{x})}$  does not depend on the size of the sample

**Question 4 (18 marks)**

- (a) Give and prove the conditions in which a systematic sample has the same precision as the corresponding stratified random sample with one unit per stratum
- (b) If  $\rho_w = \frac{E(y_{ij} - \bar{y})(y_{ik} - \bar{y})}{E(y_{ij} - \bar{y})^2} = 0$  show that the precision of systematic sampling is the same as simple random sampling with replacement. (10 marks)

**Question 5 (18 marks)**

- (a) Show how the correlation coefficient ( $\rho_{wst}$ ) between the deviations from the stratum means of pairs of items that are in the same systematic sample influence the  $V(\bar{y}_{st})$ . (10 marks)
- (b) In a population with linear trend show that systematic sample is better than simple random sampling (10 marks)