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University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3136: NUMERICAL ANALYSIS I

DATE: APRIL 2013 **TIME: $2\frac{1}{2}$HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

QUESTION ONE - (30 MARKS)

a) Given a collection of 2m data points $\{(x_i, y_i)\}_{j=0}^{2m-1}$

i. State the objective of the discrete least squares (Fourier) approximation. (1 Marks)

ii. Use determination of constants in the summation $S_n(x) = \frac{a_0}{2} + a_n \cos nx + \sum_{k=1}^{n-1}(a_k \cos kx + b_k \sin kx)$ is facilitated by orthogonality property of the functions involved. State the lemma used to show the orthogonality. (4 Marks)

b) Within rational approximations, state the two reasons that gives the Pade's method a computation advantage over the Chebyshev's method. Also state the remedies to each. (4 Marks)

c) Obtain the Pade's rational approximation of the form $\frac{a_0+a_1x}{1+b_1x}$ $to\ e^x$ (6 Marks)

d) Given that $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ determine the eigen values and eigen vectors. (7 Marks)

e) If all the eigen values of a symmetric matrix A are positive, what do we refer to such a matrix? (1 Mark)

f) Define a symmetric matrix and give an example with a 3x3 matrix. (2 Marks)

g) Consider the Chebyshev's theorem on minimax approximation.

i. State the error that should be minimized. (2 Marks)

ii. If $R^*_{mk}(x)$ is the unique rational function that minimizes e_{mk}. State the property that this rational function should satisfy. (1 ½ Marks)

iii. For $R^*_{mk}(x)$ to be minimax approximation, what condition must be satisfied by the denominator of $R^*_{mk}(x)$. (1 ½ Marks)

QUESTION TWO (20 MARKS)

a) Illustrate how the Chebyshev's polynomials are used to minimize approximation errors. (10 Marks)

b) Consider the matrix $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix}$ that has eigen values $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5$

And the eigen vector matrix $\bar{X} = [\bar{x}_1, \bar{x}_2, \bar{x}_3] = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

i. Deflate – and- reduce λ_1 using $\bar{x}_1$ and first row of A. (6 Marks)

ii. Determine the eigen values of the resulting matrix in (i) above. (2 Marks)

iii. Is deflation via the third row of A possible? Why? (2 Marks)

QUESTION THREE (20 MARKS)

a) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval [0,1] using the least squares approximation with $w(x) = 1$. (11 Marks)

b) Use Househoulder's method to transform $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ to a symmetric tridiagonal matrix. (9 Marks)

QUESTION FOUR (20 MARKS)

a) Prove that $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = 0$ when $m \neq n$. (3 Marks)

b) State the Gerschgorin's theorem. (3 Marks)

c) Use this theorem in (b) above to determine bounds for the eighenvalues of

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ -1 & 3 & 1 & 0 \\ 0 & 1 & 9 & 2 \\ 1 & 1 & 2 & 9 \end{bmatrix}$$

(6 Marks)

d) Use the power method to find the largest eigenvalue and the corresponding eigenvector (compute the first and four iterates). (8 Marks)