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University Examinations 2013/2014

FIRSTYEAR, FIRST SEMESTER EXAMINATIONS FOR DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3136: NUMERICAL ANALYSIS I

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) Given a collection of 2m data points $\{x_i, y_i\}_{i=0}^{2m-1}$

(i) Assuming that the data points are within the interval $[-\pi, \pi]$ where $x_0 = -\pi$ and $x_{2m} = \pi$, show that $x_i = -\pi + \left(\frac{i}{m}\right)\pi$ (4 Marks)

(ii) The determination of constants in the summation $s_n(x) = \frac{a_0}{2} + a_n \cos nx + \sum_{k=1}^{n-1}[a_k \cos kx + b_k \sin kx]$ is enabled by orthogonality property of the functions involved. Show that $b_k = \frac{1}{m}\sum_{i=0}^{2m-1} y_i \sin kx_i$ for $k = 1, 2, \ldots, n-1$ (6 Marks)

(b) In rational approximations of functions, state the two reasons that gives the Pade's method a computational advantage over the Chebyshev's method. Also state the improvements that can be made to the Chebyshev's method to make it better. (4 Marks)

(c) State that $f(x) \in C[a, b]$ obtain the expression for the linear normal equations that have to be solved to get $P_n(x)$, the least squares approximating polynomial for $f(x)$. (7 Marks)

(d) State the Gerschgorin's theorem. (3 Marks)

(e) Use the theorem in (d) above to determine bounds of the eigenvalues of the matrix

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ -1 & 3 & 1 & 0 \\ 0 & 1 & 9 & 2 \\ 1 & 1 & 2 & 9 \end{bmatrix}$$

(6 Marks)

QUESTION TWO – (20 MARKS)

(a) Given that $T_n(x)$ is a Chebyshev polynomial of degree $n \geq 1$

(i) Prove that $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}}\,dx = 0$ when $m \neq n$ (3 Marks)

(ii) Explain how the Chebyshev's polynomials can be used to approximate a function $f(x)$ by a lesser degree polynomial while minimizing approximation errors. (10 Marks)

(b) Given that $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ determine the eigenvalues and eigenvectors. (7 Marks)

QUESTION THREE – (20 MARKS)

(a) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval $[0,1]$ using the least squares approximation with $w(x) = 1$. (11 Marks)

(b) Find the largest eigenvalue, in magnitude, and the associated eigenvector using the power method up to the 4^{th} iterate. Use initial vector as $(1,1,1)^T$ and 4 d.p. $A = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix}$

(9 Marks)

QUESTION FOUR – (20 MARKS)

(a) Given that matrix $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix}$ has eigenvalues $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = 5$ and

eigenvector matrix $\bar{x} = [\bar{x}_1,\ \bar{x}_2,\ \bar{x}_3] = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

(i) Deflate and reduce λ_1 using $\bar{x}_1$ and first row of A. (6 Marks)

(ii) Determine the eigenvalues of the resulting matrix in (i) above. (2 Marks)

(iii)Is deflation via the third row of A possible? Why? (2 Marks)

(b) Use Householder's method to transform $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ to a symmetric tridiagonal matrix. (10 Marks)

QUESTION FIVE – (20 MARKS)

Given that $A = \begin{bmatrix} -2 & 2 & -3 \\ 3 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Obtain the matrix $P = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ that diagonalizes A.

Also determine P^{-1} i.e inverse of P. (20 Marks)