## University Examinations 2012/2013

FIRST YEAR, THIRD SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3137: NUMERICAL ANALYSIS II
DATE: DECEMBER 2012
TIME: 3HOURS
INSTRUCTIONS: Answer questions one and any other two questions

## QUESTION ONE - (30 MARKS)

a) Approximate the singular integral $\int_{0}^{\infty} e^{-x^{2}-\frac{1}{x^{2}}} d x$
b) Find the value of the integral $\int_{2}^{3} \frac{\cos 2 x}{1+\sin x} d x$, Using the Gauss-Legendre 3-point formula giving your answer correct to 6 decimal places.
c) Given the differential equation $\frac{d y}{d t}=t-y^{2}, y(0)=1$, compute $y(0.1), y(0.2)$ and $y(0.3)$ using the modified Euler method with $h=0.1$
d) Use the finite difference method to solve the boundary value problem
$\frac{d^{2} y}{d x^{2}}=y$,
$y(0)=0, y(2)=3.627$. Use $\mathrm{h}=0.5$

## QUESTION TWO - (20 MARKS)

a) Approximate the integral
$\int_{0}^{1} e^{x^{2}} d x$ Using Newton - Cotes method corresponding to $n=3$
(11 Marks)
b) Use Romberg integration method to find an approximation to the integral $\int_{0}^{3} f(x) d x$ from the following functional data:

| $x$ | 0.00 | 0.75 | 1.50 | 2.25 | 3.00 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 1.0000 | 0.47235 | 0.22313 | 0.10540 | 0.04979 |

## QUESTION THREE - (20 MARKS)

a) Estimate $\int_{0}^{0.5} \int_{0}^{0.5} \frac{\sin (x y)}{1+x y} d x d y$ Using Simpson's rule for double integrals with both steps sizes $=0.25$
b) Using Schmidt explicit formula solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}}$ Subject to the conditions $u(x, 0)=\sin \pi x, 0 \leq x \leq 1, u(0, t)=u(1, t)=0$ Carry out computation for two levels taking $h=1 / 3, k=1 / 36$.

## QUESTION FOUR - (20 MARKS)

Use the ABAM and Runge-Kutta methods to approximate $x(2)$ for the initial value problem $\frac{d x}{d t}=5 t-2 x, x(0)=1$ Using step size $h=0.5$ correct to 5 d.p.

