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University Examinations 2013/2014

FIRST YEAR, THIRD TRIMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3137: NUMERICAL ANALYSIS II

DATE: DECEMBER 2013 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

a) Determine the coefficients in the formula

$$\int_0^{2L} x^{-\frac{1}{2}} f(x)dx = (2L)^{\frac{1}{2}}\left[A_0 f(0) + A_1 f(h) + A_2 f(2h)\right],$$

such that it is exact for polynomials of as high degree as possible. (7 Marks)

b) Find the value of the integral

$I = \int_2^3 \frac{\cos 2x}{1+\sin x}dx$ using Gauss-Legendre two-point integration rule. (5 Marks)

c) Consider the initial value problem

$y' = x(y+x) - 2,\ \ y(0) = 2.$ Use the Euler method with step sizes h = 0.2 and h = 0.15 to compute approximations to y(0.6) correct to 5 decimal places. (8 Marks)

d) Evaluate the singular integral

$\int_0^1 \frac{\cos x}{\sqrt{x}}dx$. (4 Marks)

e) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, in the domain of the figure below by Gauss-Seidel method. Perform only two iterations. (6 Marks)

QUESTION TWO (20 MARKS)

a) Evaluate the double integral $\int_0^1 \int_1^2 \frac{2xy}{(1+x^2)(1+y^2)} dy\, dx$ using the Simpson's rule with h = k = 0.25. Compare your answer with the exact solution. (12 Marks)

b) Use the fourth order Runge-Kutta method to find $u = (0.2)$, of the initial value problem $\frac{du}{dt} = -2tu^2, u(0) = 1$ using h = 0.2. (8 Marks)

QUESTION THREE (20 MARKS)

a) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with conditions $u(0,t) = u(1,t) = 0, u(x,0) = \frac{1}{2}x(1-x)$and $u(x,0) = 0$, taking $h = k = 0.1$ for $0 \leq t \leq 0.2$ (12 Marks)

b) Derive the Gauss-Laguerre two-point formula and use it to evaluate the integral $\int_0^\infty \frac{e^{-x}}{1+x^2} dx$ (8 Marks)

QUESTION FOUR (20 MARKS)

a) The Lobatto quadrature formula is given by

$$\int_{-1}^{1} B_1 f(-1) + B_2 f(1) + \sum_{k=1}^{n-1} H_k f(x_k)$$

Determine B_1, B_2, x_k *and* H_k for n = 3. Find an expression for the truncation error. (11 Marks)

b) Solve the system of equations $y' = u, y(0) = 1, u' = -4y - 2u, u(0) = 1$ by the Runge – Kutta fourth order method using a step – length $h = 0.1$ at $x = 0.2$. (9 Marks)