MERU UNIVERSITY COLLEGE
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University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3130: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: DECEMBER 2012 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

QUESTION ONE - (30 MARKS)

a) Using the method of undetermined coefficients find the general solution of the differential equation

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4\sin x$$ (8 Marks)

b) Solve the initial value problem

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 2x - 1$$

$$y(0) = 1,\ \ y'(0) = -3,\ \ \ y''(0) = 4$$

c) Given the boundary value problem $x^2y'' + 2xy' + \mu y = 0, y(1) = y(e) = 0$

i. Show that it is a Sturm – Liouville problem. (3 Marks)

ii. Find the eigen values and eigen functions. (7 Marks)

d) Solve using variation of parameters method, the equation

$$\frac{d^2y}{dx^2} + y = \tan x$$ (5 Marks)

QUESTION TWO – (20 MARKS)

a) Find non-trivial solutions of the Sturm-Liouville problem

$\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0$

$y'(0)=0,\ \ y'(e^{2\pi})=0,$ where the parameter λ is non-negative. (10 Marks)

b) Solve the Bessel differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-p^2)y=0$. Where p is a non-negative constant, about the point $x=0$. (10 Marks)

QUESTION THREE – (20 MARKS)

a) Show that the function $\frac{1}{\sqrt{1-2xt+t^2}}=\sum_{n=0}^{\infty}p_n(t)\,t^n,$ where $p_n(x)$ is the Legendre polynomial of order n. (8 Marks)

b) Using the Perturbation method, solve the non-linear differential equation.

$$\frac{df}{dt}+f=\in f^2,\quad 0<\in\ll 1$$

With the initial condition $f(0)=1$ (12 Marks)

QUESTION FOUR – (20 MARKS)

a) Solve the homogeneous linear system

$$\frac{dx_1}{dt}=4x_1+3x_2+x_3$$
$$\frac{dx_2}{dt}=-4x_1-4x_2-2x_3$$
(13 Marks)
$$\frac{dx_3}{dt}=8x_1+12x_2+6x_3$$

b) Solve the Cauchy – Euler equation

$$x^3y'''(x)+x^2y''(x)-2xy'(x)+2y=0 \text{ for } x>0$$
(7 Marks)