



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## University Examinations 2012/2013

FIRST YEAR, FIRST TRIMESTER EXAMINATION FOR MASTER OF SCIENCE IN  
APPLIED MATHEMATICS

### SMA 3130: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: DECEMBER 2013

TIME: 3 HOURS

INSTRUCTIONS: Answer questions *one* and any other *two* questions

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#### QUESTION ONE (30 MARKS)

- a) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4 \sin x . \quad (8 \text{ Marks})$$

- b) Find the general solution of the linear system,

$$\begin{aligned} \frac{dx}{dt} &= 3x - 4y \\ \frac{dy}{dt} &= 2x - 3y \end{aligned} \quad (7 \text{ Marks})$$

- c) Solve the Cauchy – Euler equation

$$x^3 \frac{d^3y}{dx^3} + 9x^2 \frac{d^2y}{dx^2} + 19x \frac{dy}{dx} + 8y = 0 \quad (6 \text{ Marks})$$

- d) Use the variation of parameters methods to compute the general solution of the non-homogeneous differential equation

$$\frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 36y = e^{6x} \ln x, x > 0 \quad (9 \text{ Marks})$$

#### QUESTION TWO (20 MARKS)

- a) Find two solutions of the Bessel equation of Order  $\frac{1}{2}$  given by

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right) y = 0. \quad (10 \text{ Marks})$$

- b) Using the Perturbation method, solve the non-linear differential equation

$$\frac{dy}{dt} + y = \epsilon y^2, 0 < \epsilon \ll 1, y(0) = 1 \quad (10 \text{ Marks})$$

**QUESTION THREE (20 MARKS)**

- a) Find the general solution of the homogeneous linear system

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 8 & 12 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (13 \text{ Marks})$$

- b) Show that the function  $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} p_n(t) t^n$ , where  $p_n(x)$  is the Legendre polynomial of order  $n$ . (7 Marks)

**QUESTION FOUR (20 MARKS)**

- a) One of the solutions of the differential equation

$x^2 \frac{d^2y}{dx^2} + x^2 \frac{dy}{dx} - (x+2)y = 0, x > 0$  is  $x^{-1}e^{-x}$ . Apply the reduction of order method to obtain another linearly independent solution. (10 Marks)

- b) Solve the non-homogeneous Euler-Cauchy equation

$x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x$  by the method of variation of parameters. (10 Marks)