MERU UNIVERSITY COLLEGE
OF SCIENCE & TECHNOLOGY
P.O. Box 972-60200 Meru - Kenya. Tel: 020-2092048, 020 2069349
Fax: 020-8027449

University Examinations 2012/2013

FIRST YEAR, FIRST/THIRD SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3139: PARTIAL DIFFERENTIAL EQUATIONS 1

DATE: DECEMBER 2012 **TIME: 3HOURS**

INSTRUCTIONS: *Answer questions* ***one*** *and any other* ***two*** *questions*

QUESTION ONE - (30 MARKS)

a) Find the characteristics of the hyperbolic equation

$$x\frac{\partial^2 u}{\partial x^2} - y\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} = 0$$

Reduce the equation to the standard form and solve it. (8 Marks)

b) Use the Laplace transform to solve the equation

$$\frac{\partial u}{\partial t} = 3\frac{\partial u}{dx}, \ u(x,0) = 4e^{-2x}$$ (4 Marks)

c) Solve by separating the variables, the equation

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, \ u(0,y) = 8e^{-3y}$$ (6 Marks)

d) The steady state temperature in a semi-infinite plate is determined from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \ , 0 < x < \pi, \ y > 0$$

$$u(0,y) = 0, \ u(\pi,y) = e^{-y}, \ \ y > 0$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \ 0 < x < \pi$$

Solve for $u(x,y)$ using Fourier Cosine transform. (7 Marks)

e) For what values of x and y is the following equation hyperbolic, parabolic or Elliptic

$(y+1)\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x + y$ (5 Marks)

QUESTION TWO – (20 MARKS)

a) Using the complex form of the Fourier transform, solve the boundary value problem.

$a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, t > 0$

$u(x,0) = f(x),\ U_t(x,0) = g(x)$ (10 Marks)

b) Using the Laplace transform, solve the initial boundary value problem;

$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2},\ y(0,t) = y(5,t) = 0$

$y(x,0) = 0,\ y_t(x,0) = 5\sin \pi x$ (10 Marks)

QUESTION THREE – (20 MARKS)

a) Solve the boundary value problem

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < \infty,\ t > 0$

$u(0,t) = 0,\ t > 0$

$u(x,0) = f(x), 0 < x < \infty,$ using the appropriate Fourier transform.

(8 Marks)

b) A tightly stretched string with fixed ends $x = 0$ and $x = l$ is initially in a position given by $y = y_0 Sin^3\frac{\pi x}{l}$.

If it is released from rest from this position, find the displacement $y(x,t)$ using the separation of variables method. (12 Marks)

QUESTION FOUR – (20 MARKS)

a) Using the Eigen function expansion method, solve the initial boundary value problem.

$\frac{\partial^2 u}{dt^2} = c^2\frac{\partial^2 y}{dx^2} + \frac{e^{-t}}{\rho}, 0 < x < L,\ t > 0$

$y(0,t) = 0,\ t > 0$

$y(L,t) = 0,\ t > 0$

$y(x,0) = f(x),\ \ 0 < x < L$

$y_t(x,0) = 0,\ \ 0 < x < L$ (20 Marks)