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University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3139: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: AUGUST 2013 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) (i) Find the Fourier Sine and Cosine transforms of
$f(x) = e^{-ax}$ (8 Marks)

(ii) Using the appropriate Fourier transform, solve the equation
$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \;\; for\ x \geq 0, t \geq 0$

Under given conditions $u = u_0$ at $x = 0, t > 0$ with initial condition $u(x,0) = 0, x \geq 0$

(10 Marks)

(b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$.
(6 Marks)

(c) A semi-infinite solid $x > 0$ is initially at temperature zero. At time $t = 0$, a constant temperature u_0 is applied at the face $x = 0$. Find the temperature at any point of the solid and at any time $t > 0$, given that the temperature $u(x,t)$ is governed by the equation.

$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ (6 Marks)

QUESTION TWO – (20 MARKS)

(a) Find the Fourier series representing $f(x) = x, 0 < x < 2\pi$ and sketch its graph from $x = -4\pi\ to\ x = 4\pi.$ (8 Marks)

(b) Solve by the method of separation of variables, the following boundary value problem,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{4}\frac{\partial^2 u}{\partial t^2}$$

$u\ (0,t) = \ 0,\ u(5,t) = \ 0,\ u(x,0) = 0,\ \frac{\partial u}{\partial t}\ u(x,0) = \ 5\sin\pi x\ .$ (12 Marks)

QUESTION THREE – (20 MARKS)

(a) An infinitely long string having one end at $x = 0$ is initially at rest along x-axis. The end $x = 0$ is given a transverse displacement $f(t)$, when $t > 0$. Using the Laplace transform method find the displacement of the string at any time given the governing equation is $\frac{\partial^2 y}{\partial x^2} = c^2\frac{\partial^2 y}{\partial t^2}$ (11 Marks)

(b) Use the complex form of the Fourier transform to solve the boundary value problem

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, u(x,0) = \ f\ (x), -\infty < x < \infty$$

governing the heat conduction in a very long metal bar. (9 Marks)

QUESTION FOUR – (20 MARKS)

Use the Eigen function expansion method to solve the one-dimensional variation problem for displacement of a taut string with time-dependent forcing function,

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} + \frac{e^{-t}}{\rho}, 0 < x < l,\ t > 0$$

$y(0,t) = y(l,t) = 0,\ \ y(x,0) = f(x),\ y_t(x,0) = 0$ (20 Marks)