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University Examinations 2013/2014

FIRST YEAR, FIRST TRIMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3139: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: DECEMBER 2013 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

a) Find the characteristics of the following equation and hence reduce it to the appropriate standard form. Obtain the general solution if possible

$e^{2x}\frac{\partial^2 u}{\partial x^2}+2e^{x+y}\frac{\partial^2 u}{\partial x\partial y}+e^{2y}\frac{\partial^2 u}{\partial y^2}=0$ (11 Marks)

b) Solve using the Laplace transform the equation

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, u_x(0,t)=0,\ u(2,t)=0$

$u(x,0)=8\cos\frac{3\pi}{4}x-6\cos\frac{9\pi}{4}x$ (8 Marks)

c) Using the appropriate Fourier transform, solve the boundary value problem.

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, u(0,t)=0,\ u(x,0)=f(x)$

$0<x<\infty, t>0$ (5 Marks)

d) Find the coefficient a_n so that $1+2x=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi}{3}x$ for all x in the interval $0<x<3.$ (6 Marks)

QUESTION TWO (20 MARKS)

a) Use the method of separation of variables to solve the boundary value problem.

$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+\frac{e^{-t}}{\rho}, 0<x<50,\ t>0$

$u(0,t) = u(50,t) = 0, \quad u_t(x,0) = 0$ (8 Marks)

b) Solve the one-dimensional vibration problem for the displacement of a taut string with time – dependent forcing function $F(x,t) = e^{-t}$, given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} + \frac{e^{-t}}{\rho}$$

$$y(0,t) = y(l,t) = 0, \; y(x,0) = f(x), \; y_t(x,0) = 0$$

$$t > 0, 0 < x < l$$

Use part (a) for the initial solution. (12 Marks)

QUESTION THREE (20 MARKS)

a) Use the complex form of the Fourier transform to solve the boundary value problem:
$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, t > 0$
$u(x,0) = f(x), \; \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$ (12 Marks)

b) If $g(x) = 0$, show that the solution of part (a) can be written as
$u(x,t) = \frac{1}{2}\{f(x+at) + f(x-at)\}$. (8 Marks)

QUESTION FOUR (20 MARKS)

a) Solve using Laplace transform, the equation
$\frac{\partial^2 z}{\partial t^2} = 4\frac{\partial^2 z}{\partial x^2}, t > 0, x > 0$ where $z(0,x) = 0, \left.\frac{\partial z}{\partial x}\right|_{t=0} = 2, z(t,0) = \sin t$. (7 Marks)

b) A sheet of metal coincides with the square in the xy-plane whose vertices are the points $(0,0), (1,0), (1,1) and (0,1)$. The two faces of the sheet are perfectly insulated and the sheet is so thin that the heat flow can be regarded as two dimensional. The edges parallel to the x-axis are perfectly insulated, and the left – hand edge is maintained at a constant temperature of $0°C$. If the temperature distribution $u(1,y) = f(y)$ is maintained along the right hand edge, find the steady-state temperature distribution throughout the sheet. (13 Marks)