MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.
Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411
Fax: 064-30321
Website: www.must.ac.ke Email: info@must.ac.ke

University Examinations 2013/2014

FIRSTYEAR, FIRST SEMESTER EXAMINATIONS FOR DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3138: RIEMANNIAN GEOMETRY

DATE: APRIL 2014 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions.*

QUESTION ONE – (30 MARKS)

(a) Define the following terms:
- (i) A Hausdorff space (2 Marks)
- (ii) A differentiable manifold (2 Marks)
- (iii) Symmetric tensor (2 Marks)
- (iv) Skew symmetric tensor (2 Marks)

(b) Prove that if A_r^{pq} and B_t^s are tensors, then $C_{rt}^{pqs} = A_r^{qp} B_t^s$ is also a tensor. (4 Marks)

(c) Write the law of transformation for the tensor:

- (i) A_{jk}^i (2 Marks)
- (ii) B_{ijk}^{mn} (2 Marks)

(d) Prove that a spherical coordinate system is orthogonal. (4 Marks)

(e) Represent the vector $\vec{A} = 2i - 2xj + yk$ in cylindrical coordinates. Then determine $A_\rho,\ A_\theta,\ and\ A_z$ (5 Marks)

(f) Given that $\overline{A}^{p} = \frac{\partial \overline{x}^{p}}{\partial x^{q}} A^{q}$, prove that $A^{q} = \frac{\partial x^{q}}{d\overline{x}^{q}} \overline{A}^{p}$ (2 Marks)

(g) Prove that the contraction of the tensor A_q^p in a scalar. (3 Marks)

QUESTION TWO – (20 MARKS)

Find the square of the elements of arc-length in:

(a) Cylindrical coordinates. (5 Marks)

(b) Spherical coordinates (9 Marks)

(c) Parabolic cylindrical coordinates. (6 Marks)

QUESTION THREE – (20 MARKS)

(a) If A_r^{pq} and B_r^{pq} are tensors, prove that their sum and difference are tensors. (5 Marks)

(b) Show that the contraction of the outer product of the tensors A^p and B_q is an invariant. (5 Marks)

(c) Show that $\frac{\partial A_p}{\partial x_q}$ is not a tensor even though A_p is a covariant tensor of rank one. (5 Marks)

(d) Express the velocity $\vec{V}$ and acceleration $\vec{a}$ of a particle in cylindrical coordinates. (5 Marks)

QUESTION FOUR – (20 MARKS)

(a) Prove that $A_{p,q} - A_{q,p} = \frac{\partial A_p}{\partial x^q} - \frac{\partial A_q}{\partial x^p}$ (6 Marks)

(b) Calculate the intrinsic derivatives of

(i) an invariant ∅ (3 Marks)

(ii) A^j (5 Marks)

(iii) A_k^j (6 Marks)