University Examinations 2012/2013

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3138: RIEMANNIAN GEOMETRY

DATE: APRIL 2013   **TIME: $2\frac{1}{2}$ HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

QUESTION ONE - (30 MARKS)

a) Prove that a cylindrical coordinate system is orthogonal.   (5 Marks)

b) Represent the vector $\vec{A} = 2i - 2xj + yk$ in cylindrical coordinates. Thus determine $A_\rho, A_\emptyset$ and A_z.   (6 Marks)

c) Prove that;

    i. $\frac{d}{dt}(e_\rho) = \dot{\emptyset}\, e_\emptyset$

    ii. $\frac{d}{dt}(e_\emptyset) = \dot{\emptyset}\, e_\rho$

    Where dots denote differentiation with respect to time t.   (6 Marks)

d) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical coordinates.   (7 Marks)

e) Write the law of transformation for the tensors:

    i. A^i_{jk} (3 Marks)

    ii. B^{mn}_{ijk} (3 Marks)

QUESTION TWO (20 MARKS)

a) Find the unit vectors $e_r, e_\theta, e_\emptyset$ of spherical coordinate system in terms of i,j,k.   (5 Marks)

b) Prove that a spherical coordinate system is orthogonal.   (5 Marks)

c) Solve for i,j,k in terms of e_r, e_θ and $e_\emptyset$.. (5 Marks)

d) Represent the vector $\vec{A} = 2yi - 2zj + 3xk$ in a spherical coordinates and determine A_r, A_θ and $A_\emptyset$. (5 Marks)

QUESTION THREE (20 MARKS)

a) Determine whether $\dfrac{\partial \emptyset(x^1, x^2, \ldots, x^N)}{\partial x^k}$ is tensor. If so determine whether it is contravariant or covariant and give its rank. (8 Marks)

b) Show that $\dfrac{\partial A_p}{dx^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

(6 Marks)

c) If A_r^{pq} and B_r^{pq} are tensors, prove that their sum and difference are tensors.(6 Marks)

QUESTION FOUR (20 MARKS)

Determine the metric tensor in;

a) Cylindrical coordinates. (10 Marks)
b) Spherical coordinates. (10 Marks)