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University Examinations 2012/2013

SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 3137: NUMERICAL ANALYSIS II

DATE: AUGUST 2013 **TIME: 3 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) Perform two iterations of Picard's method to find an approximate solution of the initial value problem
$y' = x + y^2, \ y(0) = 1$ (6 Marks)

(b) Derive the expression for the error in the 3-point Gauss-Hermite formula. (9 Marks)

(c) Solve the Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$ for square mesh of the figure given below with $u(x, y) = 0$ on the boundary and mesh length, $h = 1$ (7 Marks)

(d) Define $a, b, and\ c$ such that the formula

$\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{1}{3}\right) + cf(h)\right\}$ is exact for polynomials of as high order as possible. Find the truncation error. (8 Marks)

QUESTION TWO – (20 MARKS)

(a) Find the solution $y(0.1)$ of the initial value problem $y' = -2ty^2, y(0) = 1$ with $h = 0.1$, using Runge-Kutta method of order four. (5 Marks)

(b) Find the values of $u(x,t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{1}{2}x^2$ at the points: $x = i$: $i = 0, 1, 2, \ldots 7$ and $t = \frac{1}{8}j : j = 0, 1, 2.$ (15 Marks)

QUESTION THREE – (20 MARKS)

(a) Use Runge-Kutta method to find $y(0.2)$ for the equation $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y$, given that $y = 1, \frac{dy}{dx} = 0$ when $x = 0$. (10 Marks)

(b) (i) Using the method of undetermined coefficients, find the nodes and the weights of the quandrature formula.

$$\int_0^\infty e^{-x} f(x)dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$$

(ii) Evaluate the integral $\int_0^\infty \frac{e^{-x}}{1+x^2}dx$ using the method derived in (i). (10 Marks)

QUESTION FOUR– (20 MARKS)

(a) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with conditions $u(0,t) = u(1,t) = 0,\ u(x,0) = \frac{1}{2}x(1-x)$ and $u(x,0) = 0,$ taking $h = k = 0.1$ for $0 \le t \le 0.4$. (11 Marks)

(b) Find the quandrature formula

$\int_0^1 \frac{f(x)}{\sqrt{x(1-x)}}\, dx = \propto_1 f(0) + \propto_2 f\left(\frac{1}{2}\right) + \propto_3 f(1)$ which is exact for polynomial of highest possible degree. Use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (9 Marks)